

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Instruction – Transition Task

- Aim to complete this booklet independently
- If you need support, use the video/ written solutions provided on the Urmston Grammar website.
- Complete all exam questions at the end of each section and mark them using the mark scheme provided.
- You do not need to do anything with the exercise boxes ->

Exercise 1A Page 3

First few lessons at Urmston Grammar

Lesson 1 – You will hand in your transition work to your teacher and then revise chapters 1, 2 and 3 in preparation for your skills test.

Lesson 2 – You will complete a skills test on chapters 1, 2, and 3

Lesson 3 – You will start new content.

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Diagnostic for Chapter 1 Algebraic Expressions

1 Simplify: a $4m^2n + 5mn^2 - 2m^2n + mn^2 - 3mn^2$ b $3x^2 - 5x + 2 + 3x^2 - 7x - 12$ ← GCSE Mathematics 2 Write as a single power of 2: a $2^5 \times 2^3$ b $2^6 \div 2^2$ c $(2^3)^2$ ← GCSE Mathematics a) $2m^2n + 3mn^2$ b) $6x^2 - 12x - 10$ 2 a) 2^8 b) 2^4 c) 2^6	3 Expand: a $3(x + 4)$ b $5(2 - 3x)$ c $6(2x - 5y)$ ← GCSE Mathematics a) $30x + 12$ b) $10 - 15x$ c) $12x - 30y$																
4 Write down the highest common factor of: a 24 and 16 b $6x$ and $8x^2$ c $4xy^2$ and $3xy$ ← GCSE Mathematics a) <table> <tr> <td>1</td><td>24</td> <td>1</td><td>16</td> </tr> <tr> <td>2</td><td>12</td> <td>2</td><td>8</td> </tr> <tr> <td>3</td><td>8</td> <td>4</td><td>4</td> </tr> <tr> <td>4</td><td>6</td> <td></td><td></td> </tr> </table> HCF = 8 b) <u>$2x$</u> c) <u>xy</u>	1	24	1	16	2	12	2	8	3	8	4	4	4	6			5 Simplify: a $\frac{10x}{5}$ b $\frac{20x}{2}$ c $\frac{40x}{24}$ a) $2x$ b) $10x$ c) $\frac{10}{6}x = \frac{5}{3}x$
1	24	1	16														
2	12	2	8														
3	8	4	4														
4	6																

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Basic Index Laws

$$(a^n)^m = a^{n \times m} = a^{nm} \quad \parallel \quad a^n \div a^m = a^{n-m}$$

$$a^n \times a^m = a^{n+m}$$

Examples

2. Simplify $(a^3)^2 \times 2a^2$

$$= a^6 \times 2a^2$$

$$= 2a^8$$

2. Simplify $(4x^3y)^3$

$$= 4^3 (x^3)^3 y^3$$

$$= 64x^9y^3$$

Alt

$$4x^3y \times 4x^3y \times 4x^3y$$

$$64x^9y^3$$

3. Simplify $2x^2(3 + 5x) - x(4 - x^2)$

$$6x^2 + 10x^3 - 4x + x^3$$

$$= 11x^3 + 6x^2 - 4x$$

4. Simplify $\frac{x^3 - 2x}{3x^2}$ (2 methods)

$$\frac{\cancel{x}(x^2 - 2)}{3\cancel{x}^2} = \frac{x^2 - 2}{3x} \quad \parallel \quad \text{Alt}$$

$$\frac{x^3}{3x^2} - \frac{2x}{3x^2}$$

$$\frac{x}{3} - \frac{2}{3x}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Test Your Understanding:

1. Simplify $\left(\frac{2a^5}{a^2}\right)^2 \times 3a = \frac{4a^{10}}{a^4} \times 3a.$

$$= \frac{12a^{11}}{a^4}$$

$$= \underline{\underline{12a^7}}$$

2. Simplify $\frac{2x+x^5}{4x^3}$

$$= \frac{x(2+x^4)}{4x^3}$$

$$= \frac{2+x^4}{4x^2}$$

$$= \frac{2+x^4}{4x^2}$$

3. Expand and simplify $2x(3-x^2) - 4x^3(3-x)$

$$6x - 2x^3 - 12x^3 + 4x^4$$

$$= \underline{\underline{4x^4 - 14x^3 + 6x}}$$

4. Simplify $\underline{\underline{2^x \times 3^x}}$

$$2 \times 2 \times 2 \dots \times 2 \times 3 \times 3 \times 3 \dots \times 3$$

$$= 2 \times 3 \times 2 \times 3 \times 2 \times 3 \dots \times 2 \times 3$$

$$= \underline{\underline{6^x}}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Negative and Fractional Indices

$a^{-1} = \frac{1}{a}$	$a^{1/2} = \sqrt{a}$	$a^{b/c} = \sqrt[c]{a^b}$
$a^{-b} = \frac{1}{a^b}$	$a^{1/b} = \sqrt[b]{a}$	

1. Prove that $x^{\frac{1}{2}} = \sqrt{x}$

$$x^{1/2} \times x^{1/2} = x^1$$

$$x^{1/2} = \sqrt{x}$$

2. Evaluate $27^{-1/3}$

$$27^{-1/3} = \left(\frac{1}{27}\right)^{1/3}$$

$$= \sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}}$$

$$= \frac{1}{3}$$

3. Evaluate $32^{\frac{2}{5}}$

$$\sqrt[5]{32^2} = 2^2$$

$$= 4$$

4. Simplify $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

$$\sqrt{\frac{1}{9}x^6y} = \frac{1}{3}x^3y^{1/2}$$

2. Evaluate $\left(\frac{27}{8}\right)^{-2/3}$

$$= \left(\frac{8}{27}\right)^{2/3}$$

$$= \left(\frac{2}{3}\right)^2$$

$$\frac{4}{9}$$

6. If $b = \frac{1}{9}a^2$, determine $3b^{-2}$ in the form ka^n where k, n are constants

$$3\left(\frac{1}{9}a^2\right)^{-2}$$

$$= 3\left(\frac{a^2}{9}\right)^{-2}$$

$$= 3\left(\frac{9}{a^2}\right)^2$$

$$= 3\left(\frac{81}{a^4}\right)$$

$$= \frac{243}{a^4}$$

$$= \underline{\underline{243a^{-4}}}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Brackets: Expanding

Example: $(x + 1)(x + 2)(x + 3)$

$$(x^2 + 2x + x + 2)(x + 3)$$

$$(x^2 + 3x + 2)(x + 3)$$

$$x^3 + 3x^2 + 2x + 3x^2 + 9x + 6$$

$$x^3 + 6x^2 + 11x + 6$$

Questions

2. Expand and simplify

$$(x + 5)(x - 2)(x + 1)$$

$$(x^2 + 3x - 10)(x + 1)$$

$$x^3 + 3x^2 - 10x + x^2 + 3x - 10$$

$$x^3 + 4x^2 - 7x - 10$$

2. Expand and simplify:

$$2(x - 3)(x - 4)$$

$$2(x^2 - 7x + 12)$$

$$2x^2 - 14x + 24$$

2. Expand and simplify:

$$(2x - 1)^3$$

$$(4x^2 - 4x + 1)(2x - 1)$$

$$8x^3 - 8x^2 + 2x - 4x^2 + 4x - 1$$

$$8x^3 - 12x^2 + 6x - 1$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Brackets: Factorising

Examples:

$$1. x^2 - 5x - 14 \quad \begin{matrix} + & - & 5 \\ \times & - & 14 \end{matrix}$$

$$(x - 7)(x + 2)$$

$$2. 2x^2 + 5x - 12 \quad \begin{matrix} M & A & M \end{matrix}$$

$$(2x - 3)(x + 4)$$

$$\begin{matrix} \otimes & -24 & +8 \\ \oplus & 5 & -3 \end{matrix}$$

	$2x$	$x + 4$
$2x$	$2x^2$	$8x$
-3	$-3x$	-12

$$3. 4x^2 - 9$$

$$(2x + 3)(2x - 3)$$

$$4. x^3 - x$$

$$x(x^2 - 1)$$

$$x(x - 1)(x + 1)$$

$$2. x^3 + 3x^2 + 2x$$

$$x(x^2 + 3x + 2)$$

$$x(x + 2)(x + 1)$$

Test your understanding:

Factorise completely

$$1. 6x^2 + x - 2$$

$$(3x + 2)(2x - 1)$$

$$2. x^3 - 7x^2 + 12x$$

$$x(x^2 - 7x + 12)$$

$$x(x - 3)(x + 4)$$

$$3. x^4 - 1$$

$$(x^2 - 1)(x^2 + 1)$$

$$(x - 1)(x + 1)(x^2 + 1)$$

$$4. x^3 - 1$$

$$(x - 1)(x^2 + x + 1)$$

↑
at this stage
we do not need
to know this.

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Surds:

Recap:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$2 \times \sqrt{a} = 2\sqrt{a}$$

$$\sqrt{a^2} = a$$

Simplify:

$$1. \sqrt{3} \times 2 = 2\sqrt{3}$$

$$2. 3\sqrt{5} \times 2\sqrt{5} \quad 6 \times 5 \\ = 30$$

$$3. \sqrt{8} = \sqrt{4} \times \sqrt{2} \\ = 2\sqrt{2}$$

$$4. \sqrt{12} + \sqrt{27} = \sqrt{4} \times \sqrt{3} + \sqrt{9} \times \sqrt{3} \\ = 2 \times \sqrt{3} + 3 \times \sqrt{3} \\ = 5\sqrt{3}$$

$$2. (\sqrt{8} + 1)(\sqrt{2} - 3)$$

$$= \sqrt{16} + \sqrt{2} - 3\sqrt{8} - 3$$

$$= 4 + \sqrt{2} - 3(2\sqrt{2}) - 3$$

$$= 1 + \sqrt{2} - 6\sqrt{2}$$

$$= 1 - 5\sqrt{2}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Rationalising the denominator:

Examples:

$$1. \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

$$2. \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

$$3. \frac{7}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{7\sqrt{7}}{7} = \sqrt{7}$$

$$4. \frac{15}{\sqrt{5}} + \sqrt{5} = \frac{15}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} + \sqrt{5} = \frac{15\sqrt{5}}{5} + \sqrt{5}$$

Test your understanding:

$$\frac{12}{\sqrt{3}} = \frac{12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$3\sqrt{5} + \sqrt{5} = \underline{\underline{4\sqrt{5}}}$$

$$\frac{2}{\sqrt{6}} = \frac{2 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$$

$$\frac{4\sqrt{2}}{\sqrt{8}} = \frac{4\sqrt{2}}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{4\sqrt{16}}{8} = \frac{4 \times 4}{8} = \frac{16}{8} = \underline{\underline{2}}$$

$$\text{Alt } \frac{4\sqrt{2}}{2\sqrt{2}} = \underline{\underline{2}}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

More Complicated Examples:

$$1. \frac{1}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \frac{\sqrt{2}-1}{2 + \sqrt{2} - \sqrt{2} - 1}$$

$$= \frac{\sqrt{2}-1}{1}$$

$$= \underline{\underline{\sqrt{2}-1}}$$

$$2. \frac{3}{(\sqrt{6}-2)} \times \frac{(\sqrt{6}+2)}{(\sqrt{6}+2)}$$

$$= \frac{3\sqrt{6} + 6}{6 - 4}$$

$$= \frac{3\sqrt{6} + 6}{2}$$

$$\frac{3\sqrt{6}}{2} + 3$$

$$3. \frac{4}{\sqrt{3}+1} = \frac{4(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{4\sqrt{3}-4}{3-1}$$

$$= \frac{4\sqrt{3}}{2} - \frac{4}{2}$$

$$= \underline{\underline{2\sqrt{3}-2}}$$

$$4. \frac{3\sqrt{2}+4}{5\sqrt{2}-7} = \frac{(3\sqrt{2}+4)(5\sqrt{2}+7)}{(5\sqrt{2}-7)(5\sqrt{2}+7)}$$

$$= \frac{15(2) + 20\sqrt{2} + 21\sqrt{2} + 28}{25(2) - 49}$$

$$= \frac{30 + 20\sqrt{2} + 21\sqrt{2} + 28}{1}$$

$$= 58 + 41\sqrt{2}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Test Your Understanding:

Rationalise the denominator and simplify

$$2. \frac{4}{\sqrt{5}-2} \times \frac{(\sqrt{5}+2)}{(\sqrt{5}+2)} = \frac{4\sqrt{5}+8}{5-4} = 4\sqrt{5}+8$$

$$2. \frac{2\sqrt{3}-1}{3\sqrt{3}+1} \times \frac{(2\sqrt{3}-1)(3\sqrt{3}-1)}{(3\sqrt{3}+1)(3\sqrt{3}-1)} = \frac{6(3) - 3\sqrt{3} - 2\sqrt{3} + 1}{9(3) - 1} = \frac{18 - 5\sqrt{3} + 1}{26} = \frac{19 - 5\sqrt{3}}{26}$$

2. Solve $y(\sqrt{3}-1) = 8$

Give your answer in the form $a + b\sqrt{3}$ where a and b are integers.

$$y = \frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{8\sqrt{3}+8}{3-1} = \frac{8\sqrt{3}+8}{2} = 4\sqrt{3}+4$$

Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.



Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Exam Questions – For revision purposes

Q1.

(a) Simplify

$$\sqrt{32} + \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Simplify

$$\frac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}}$$

giving your answer in the form $b\sqrt{2} + c$, where b and c are integers.

(4)

Total 6 marks

Q2.

(a) Find the value of $16^{\frac{1}{4}}$

(2)

(b) Simplify $x(2x^{-\frac{1}{4}})^4$

(2)

(Total 4 marks)

Q3.

Simplify

$$\frac{5 - 2\sqrt{3}}{\sqrt{3} - 1}$$

giving your answer in the form $p + q\sqrt{3}$, where p and q are rational numbers.

(4)

(Total 4 marks)

Q4.

(a) Evaluate $(32)^{\frac{3}{5}}$, giving your answer as an integer.

(2)

(b) Simplify fully $\left(\frac{25x^4}{4}\right)^{\frac{1}{2}}$

(2)

(Total 4 marks)

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Q5.

(a) Expand and simplify $(7 + \sqrt{5})(3 - \sqrt{5})$

(3)

(b) Express $\frac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

(3)

(Total 6 marks)

Q6.

Simplify

(a) $(3\sqrt{7})^2$

(1)

(b) $(8 + \sqrt{5})(2 - \sqrt{5})$

(3)

(Total 4 marks)

Q7.

Given that $32\sqrt{2} = 2^a$, find the value of a .

(3)

(Total 3 marks)

Q8.

(2) Express

$$(5 - \sqrt{8})(1 + \sqrt{2})$$

in the form $a + b\sqrt{2}$, where a and b are integers.

(3)

(ii) Express

$$\sqrt{80 + \frac{30}{\sqrt{5}}}$$

in the form $c\sqrt{5}$, where c is an integer.

(3)

(Total 6 marks)

Q9.

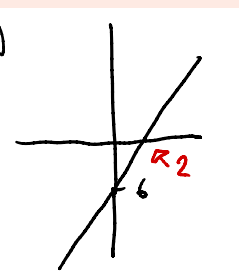
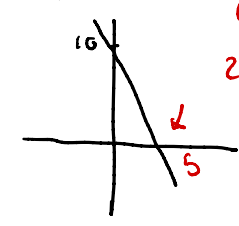
Express 8^{2x+3} in the form 2^y , stating y in terms of x .

(2)

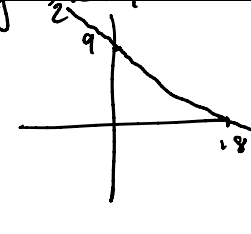
(Total 2 marks)

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Diagnostic for Chapter 2 Quadratics

1 Solve the following equations: a $3x + 6 = x - 4$ b $5(x + 3) = 6(2x - 1)$ c $4x^2 = 100$ d $(x - 8)^2 = 64$ ← GCSE Mathematics a) $3x + 6 = x - 4$ $2x = -10$ $x = -5$ b) $5x + 15 = 12x - 6$ $21 = 7x$ $3 = x$ c) $4x^2 = 100$ Alt $2x = \pm 10$ $x^2 = 25$ $x = \pm 5$ d) $(x - 8)^2 = 64$ $x - 8 = \pm 8$ $x - 8 = 8$ $x - 8 = -8$ $x = 16$ $x = 0$	2 Factorise the following expressions: a $x^2 + 8x + 15$ b $x^2 + 3x - 10$ c $3x^2 - 14x - 5$ d $x^2 - 400$ a) $(x + 3)(x + 5)$ d) $x^2 - 400$ b) $(x + 5)(x - 2)$ $(x - 20)(x + 20)$ c) $\begin{matrix} \textcircled{+} & -15 & \textcircled{+} & -15 \\ \textcircled{+} & -14 & \textcircled{+} & +1 \end{matrix}$ $\begin{matrix} x & -5 \\ 3x & -15x \\ 1 & -5 \end{matrix}$ $(3x + 1)(x - 5)$
3 Sketch the graphs of the following equations, labelling the points where each graph crosses the axes: a $y = 3x - 6$ b $y = 10 - 2x$ c $x + 2y = 18$ d $y = x^2$ ← GCSE Mathematics a)  b)  c) $x + 2y = 18$ $\Rightarrow 2y = -x + 18$ $y = -\frac{1}{2}x + 9$ d) 	4 Solve the following inequalities: a $x + 8 < 11$ b $2x - 5 \geq 13$ c $4x - 7 \leq 2(x - 1)$ d $4 - x < 11$ ← GCSE Mathematics When you divide or multiply by a negative integer. Remember to flip the sign. a) $x + 8 < 11$ c) $4x - 7 \leq 2x - 2$ $x < 3$ $2x \leq 5$ b) $2x - 5 \geq 13$ $x \leq 2.5$ $2x \geq 18$ d) $4 - x < 11$ $x \geq 9$ $-x < 7$ $\underline{x \geq -7}$ Alt $4 < 11 + x$ $-7 < x$ $x > -7$

c) $x + 2y = 18$
 $\Rightarrow 2y = -x + 18$
 $y = -\frac{1}{2}x + 9$
 $x = 0$ $0 + 2y = 18$
 $2y = 18$
 $y = 9$
 $(0, 9)$
 $y = 0$ $x + 2(0) = 18$
 $x = 18$
 $(18, 0)$



Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Solving Quadratic Equations

The 3 ways to solve a quadratic:

→ factorisation

→ formula

→ completing the square

Recap:

By factorisation

$$1. x^2 + 5x - 6 = 0$$

$$(x+6)(x-1) = 0$$

$$x+6=0 \quad \text{or} \quad x-1=0$$

$$x = -6 \quad x = 1$$

$$\underline{x = -6} \quad \underline{x = 1}$$

Using the Quadratic Formula

$$2. x^2 + 5x - 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 + 24}}{2}$$

$$x = \frac{-5 \pm \sqrt{49}}{2}$$

$$a = 1$$

$$b = 5$$

$$c = -6$$

$$x = \frac{-5 \pm 7}{2}$$

$$\oplus \quad x = \frac{2}{2} = 1$$

$$\ominus \quad x = \frac{-12}{2} = -6$$

Examples

$$1. (x-1)^2 = 5$$

$$x-1 = \pm\sqrt{5}$$

$$x = 1 \pm \sqrt{5}$$

$$2. \text{ Solve } x - 6\sqrt{x} + 8 = 0$$

$$\text{let } y = \sqrt{x}$$

$$y^2 - 6y + 8 = 0$$

$$(y-4)(y-2) = 0$$

$$y = 4 \quad y = 2$$

$$\sqrt{x} = 4 \quad \sqrt{x} = 2$$

$$x = 16 \quad x = 4$$

$$(\sqrt{x} - 4)(\sqrt{x} - 2) = 0$$

$$\sqrt{x} = 4 \quad \sqrt{x} = 2$$

$$\underline{x = 16} \quad \underline{x = 4}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Test your understanding

1. $(x + 3)^2 = x + 5$

$$x^2 + 6x + 9 = x + 5$$

$$x^2 + 5x + 4 = 0$$

$$(x + 4)(x + 1) = 0$$

$$x = -4 \quad x = -1$$

2. $(2x + 1)^2 = 5$

$$2x + 1 = \pm\sqrt{5}$$

$$2x = -1 \pm \sqrt{5}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

3. $\sqrt{x + 3} = x - 3$

$$x + 3 = (x - 3)^2$$

$$x + 3 = x^2 - 6x + 9$$

$$0 = x^2 - 7x + 6$$

$$0 = (x - 6)(x - 1)$$

$$x = 6 \quad x = 1$$

4. $2x + \sqrt{x} - 1 = 0$

let $y = \sqrt{x}$
 $y^2 = x$

$$2y^2 + y - 1 = 0$$

$$(2y - 1)(y + 1) = 0$$

$$\begin{aligned} 2y &= 1 & y &= -1 \\ y &= \frac{1}{2} & \sqrt{x} &= -1 \\ \sqrt{x} &= \frac{1}{2} & x &= 1 \\ x &= \frac{1}{4} \end{aligned}$$

Exercise 2A/ 2B Page
20/ 22

Solving by Completing the Square

Completing the Square form:

$$= d(x + a)^2 + b$$

Worked Examples (a = 1): $ax^2 + bx + c$

1. $x^2 + 12x$

$$(x + 6)^2 - 36$$

$$-(-1)^2$$

3. $x^2 - 2x$

$$(x - 1)^2 - 1$$

2. $x^2 + 8x$

$$(x + 4)^2 - 16$$

4. $x^2 - 6x + 7$

$$(x - 3)^2 - 9 + 7$$

$$(x - 3)^2 - 2$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

More complicated examples (a not equal to 1):

1. Express $2x^2 + 12x + 7$ in the form $a(x + b)^2 + c$

$$\begin{aligned} 2 \left[x^2 + 6x \right] + 7 \\ 2 \left[(x + 3)^2 - 9 \right] + 7 \\ 2(x + 3)^2 - 18 + 7 \\ 2(x + 3)^2 - 11 \end{aligned}$$

2. Express $5 - 3x^2 + 6x$ in the form $a - b(x + c)^2$

$$\begin{aligned} -3x^2 + 6x + 5 \\ -3 \left[x^2 - 2x \right] + 5 \\ -3 \left[(x - 1)^2 - 1 \right] + 5 \\ -3(x - 1)^2 + 3 + 5 \end{aligned} \quad \rightarrow \quad 8 - 3(x - 1)^2$$

Test Your Understanding:

1. Express $3x^2 - 18x + 4$ in the form $a(x + b)^2 + c$

$$\begin{aligned} 3 \left[x^2 - 6x \right] + 4 \\ 3 \left[(x - 3)^2 - 9 \right] + 4 \\ 3(x - 3)^2 - 27 + 4 \\ 3(x - 3)^2 - 23 \end{aligned}$$

2. Express $20x - 5x^2 + 3$ in the form $a - b(x + c)^2$

$$\begin{aligned} -5x^2 + 20x + 3 \\ -5 \left[x^2 - 4x \right] + 3 \\ -5 \left[(x - 2)^2 - 4 \right] + 3 \end{aligned} \quad \rightarrow \quad \begin{aligned} -5(x - 2)^2 + 20 + 3 \\ -5(x - 2)^2 + 23 \end{aligned}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Solving by Completing the Square:

Note: Previously we factorised out the 3. This is because $3x^2 - 18x + 4$ on its own is an **expression**, so dividing by 3 (instead of factorising) would change the expression. However, in an equation, we can divide both sides by 3 without affecting the solutions.

Example

Solve the equation $3x^2 - 18x + 4 = 0$ by completing the square.

Completing the square

$$3(x^2 - 6x) + 4 = 0$$

$$3[(x - 3)^2 - 9] + 4 = 0$$

$$3(x - 3)^2 - 27 + 4 = 0$$

$$3(x - 3)^2 - 23 = 0$$

Solving.

$$3(x - 3)^2 = 23$$

$$(x - 3)^2 = \frac{23}{3}$$

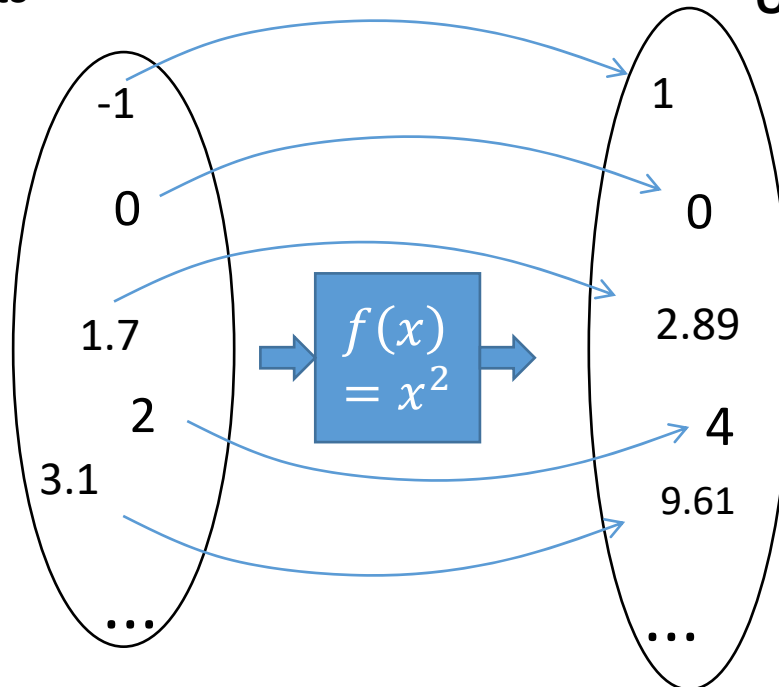
$$x - 3 = \pm \sqrt{\frac{23}{3}}$$

$$x = 3 \pm \sqrt{\frac{23}{3}}$$

Functions:

Inputs

Outputs



$$f(x) = y$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 - Quadratics. Chapter 3 - Equations and Inequalities

Examples:

1. If $f(x) = x^2 - 3x$ and $g(x) = x + 5$, $x \in \mathbb{R}$

a) Find $f(-4)$

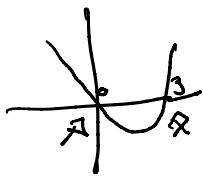
b) Find the values of x for which $f(x) = g(x)$

c) Find the roots of $f(x)$. $\rightarrow$ cross x -axis on your graph

d) Find the roots of $g(x)$.

$$\begin{aligned} \text{a)} \quad f(-4) &= (-4)^2 - 3(-4) \\ &= 16 + 12 \\ &= \underline{\underline{28}} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad x^2 - 3x &= x + 5 \\ x^2 - 4x - 5 &= 0 \\ (x - 5)(x + 1) &= 0 \\ x &= 5 \quad x = -1 \end{aligned}$$

$$\begin{aligned} \text{b)} \quad f(x) &= x^2 - 3x \\ f(x) &= 0 \\ 0 &= x^2 - 3x \\ 0 &= x(x - 3) \\ x &= 0 \quad x = 3 \rightarrow \text{roots} \end{aligned}$$


$$\begin{aligned} \text{d)} \quad g(x) &= x + 5 \\ g(x) &= 0 \\ 0 &= x + 5 \\ x &= \underline{\underline{-5}} \end{aligned}$$

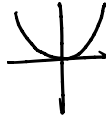
2. Determine the minimum value of the function $f(x) = x^2 - 6x + 2$, and state the value of x for which this minimum occurs.

$$\begin{aligned} f(x) &= x^2 - 6x + 2 \\ &= (x - 3)^2 - 9 + 2 \\ &= (x - 3)^2 - 7 \end{aligned}$$

$$\min \quad \underline{\underline{(3, -7)}}$$

$$\begin{aligned} \min \quad (x + a)^2 + b \\ (-a, b) \end{aligned}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

$$y = x^2$$


$$y = -x^2$$


Test Your Understanding:

$f(x)$	Completed square	Min/max value of $f(x)$	x for which this min/max occurs
$x^2 + 4x + 9$	$(2x + 2)^2 + 5$	$(-2, 5)$ min	-2
$x^2 - 10x + 21$	$(x - 5)^2 - 4$	$(5, -4)$ min	5
$10 - x^2$	$-(x + 0)^2 + 10$	$(0, 10)$ max	0
$8 - x^2 + 6x$ $-(x^2 - 6x) + 8$	$-[(x - 3)^2 - 9] + 8$ $-(x - 3)^2 + 9 + 8$ $-(x - 3)^2 + 17$	$(3, 17)$ max	3

1. Find the minimum value of $f(x) = 2x^2 + 12x - 5$ and state the value of x for which this occurs.

$$\begin{aligned}
 f(x) &= 2[x^2 + 6x] - 5 \\
 &= 2[(x + 3)^2 - 9] - 5 \\
 &= 2(x + 3)^2 - 18 - 5 \\
 &= 2(x + 3)^2 - 23
 \end{aligned}$$

$$\text{min } (-3, -23)$$

$$\underline{\underline{x = -3}}$$

2. Find the roots of the function $f(x) = 2x^2 + 3x + 1$

$$f(x) = (2x + 1)(x + 1)$$

$$x = -\frac{1}{2} \quad x = -1$$

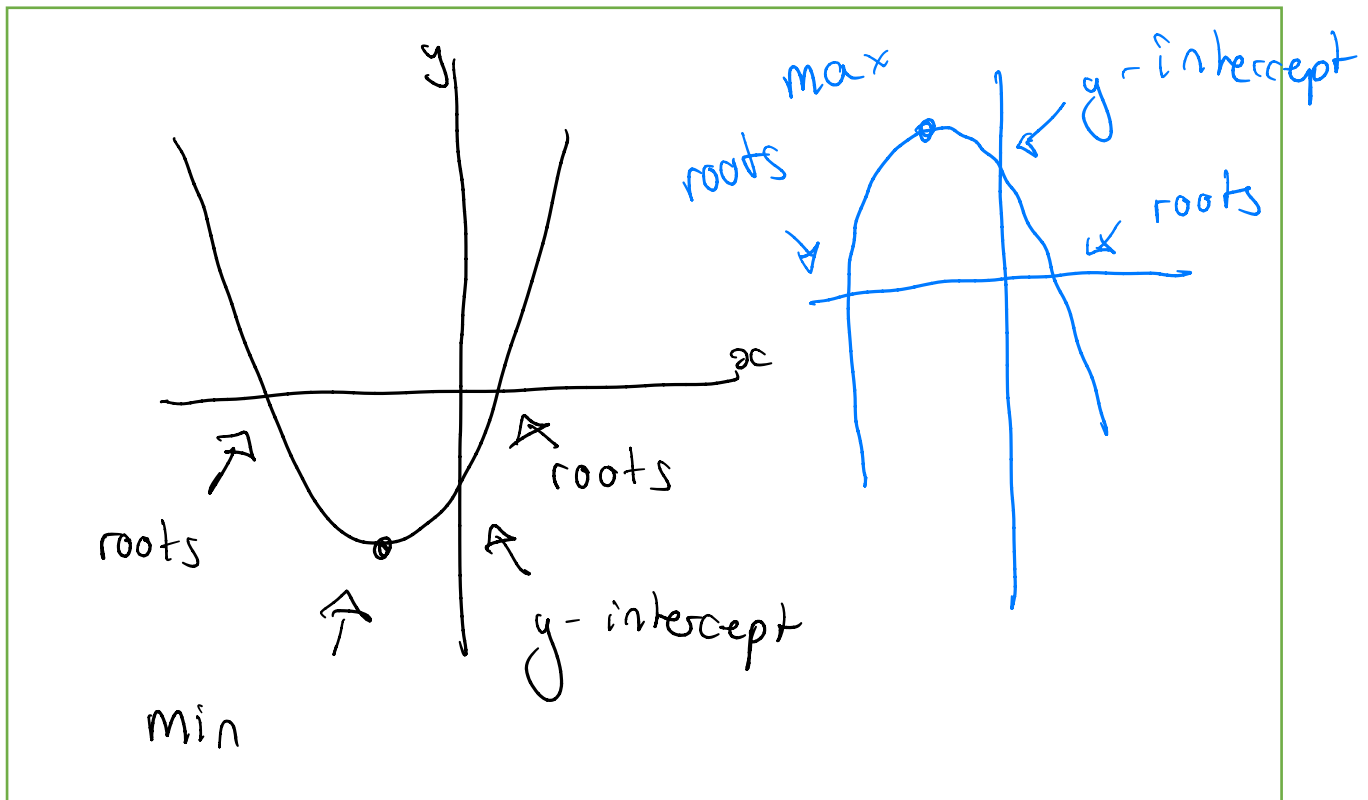
3. Find the roots of the function $f(x) = x^4 - x^2 - 6$

$$\begin{aligned}
 y &= x^2 & \sqrt{y} &= x & f(\sqrt{y}) &= y^2 - y - 6 \\
 y^2 &= x^4 & & & &= (y - 3)(y + 2) \\
 & & y &= 3 & y &= -2 \\
 & & x^2 &= 3 & x^2 &= -2 \\
 & & x &= \pm\sqrt{3} & & \times
 \end{aligned}$$

Exercise 2E Page 27

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Quadratic Graphs:



Example: Sketch the graph of $y = x^2 + 3x - 4$ and find the coordinates of the turning point.

$$y = x^2 + 3x - 4$$

$$y = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 4$$

$$y = \left(x + \frac{3}{2}\right)^2 - \frac{25}{4}$$

min $\left(-\frac{3}{2}, -\frac{25}{4}\right)$

$\uparrow (0, -4)$

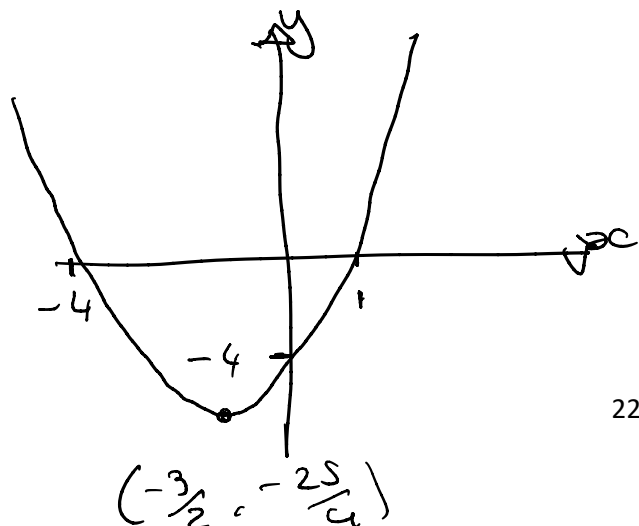
$$-\frac{9}{4} - \frac{16}{4}$$

$$-\frac{25}{4}$$

find roots $y = x^2 + 3x - 4$

$$0 = (x + 4)(x - 1)$$

$$x = -4 \quad x = 1$$



Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Example

↙ ↘ (0, -3)

Sketch the graph of $y = 4x - 2x^2 - 3$ and find the coordinates of the turning point. Write down the equation of the line of symmetry.

$$y = -2x^2 + 4x - 3$$

$$y = -2[x^2 - 2x] - 3$$

$$y = -2[(x-1)^2 - 1] - 3$$

$$y = -2(x-1)^2 + 2 - 3$$

$$y = -2(x-1)^2 - 1$$

$$\text{max } (1, -1)$$

Root

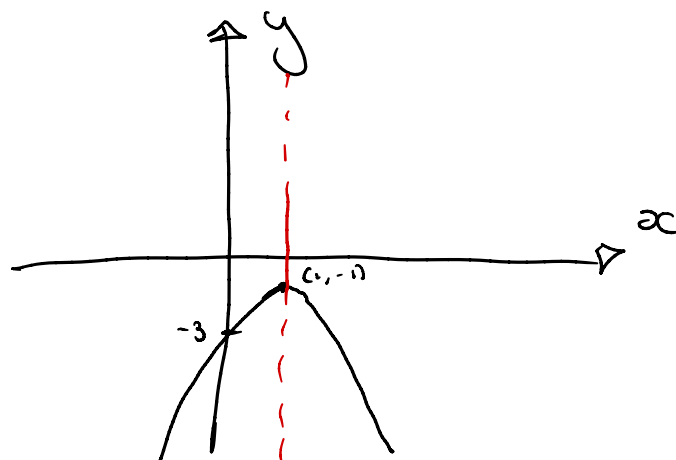
$$0 = -2(x-1)^2 - 1$$

$$1 = -2(x-1)^2$$

$$\frac{1}{-2} = (x-1)^2$$

$$\sqrt{\frac{1}{-2}} = \text{X error.}$$

no roots

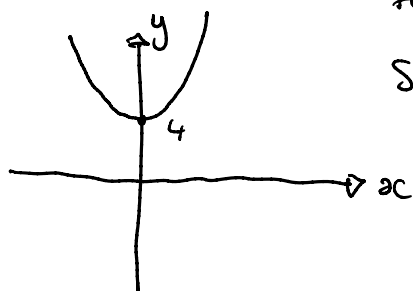


$x = 1 \Rightarrow$ line of symmetry

Test Your Understanding

Sketch the following, indicating any intercepts with the axis, the turning point and the equation of the line of symmetry.

1. $y = x^2 + 4$



turning point = (0, 4)

Symmetry $x = 0$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

$$2. y = x^2 - 7x + 10$$

$$y = (x-5)(x-2)$$

$$x=5 \quad x=2$$

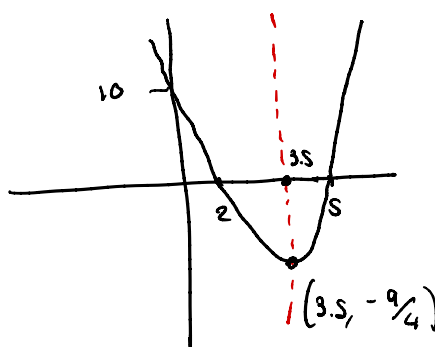
line of symmetry $\Rightarrow x = 3.5$

min point

$$y = (3.5)^2 - 7(3.5) + 10$$

$$y = -\frac{9}{4}$$

$$(3.5, -\frac{9}{4})$$



$$\frac{5+2}{2} = 3.5$$

$$3. y = 5x + 3 - 2x^2$$

$$y = -2x^2 + 5x + 3$$

$$y = -2\left[x^2 - \frac{5}{2}x\right] + 3$$

$$y = -2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] + 3$$

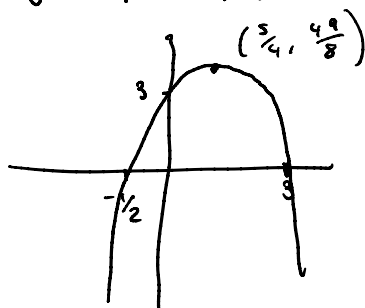
$$y = -2\left(x - \frac{5}{4}\right)^2 + \frac{23}{8} + 3$$

$$y = -2\left(x - \frac{5}{4}\right)^2 + \frac{49}{8}$$

$$\left(\frac{5}{4}, \frac{49}{8}\right) \text{ max}$$

from calc $\Rightarrow x = 3 \quad x = -\frac{1}{2}$

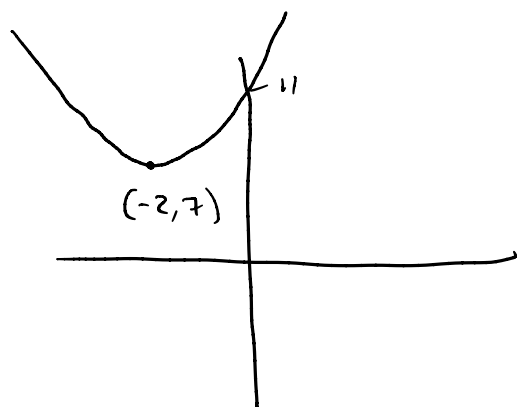
line of symmetry $x = \frac{5}{4}$
y intercept $(3, 0)$



Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

4. $y = x^2 + 4x + 11$

from calc. $x = -2 + \sqrt{7}i \rightarrow$ complex number
no roots.



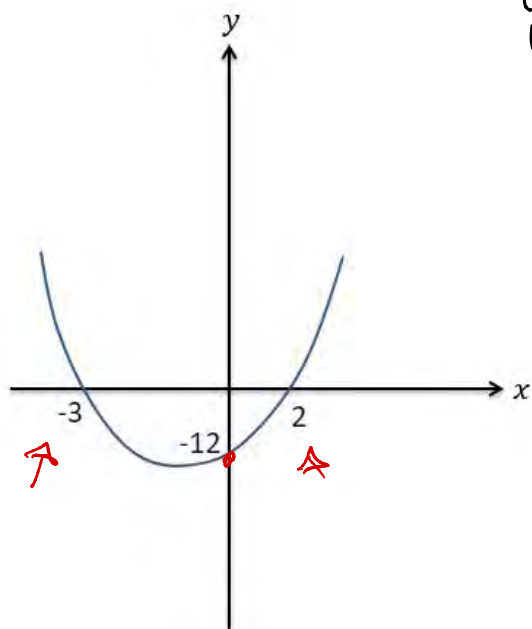
$(-2, 7)$

Complete.

$$y = (x+2)^2 - 4 + 11$$

$$y = \underline{\underline{(x+2)^2 + 7}}$$

Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



$$y = 2(x+3)(x-2)$$

$$x^2 + 3x - 2x - 6$$

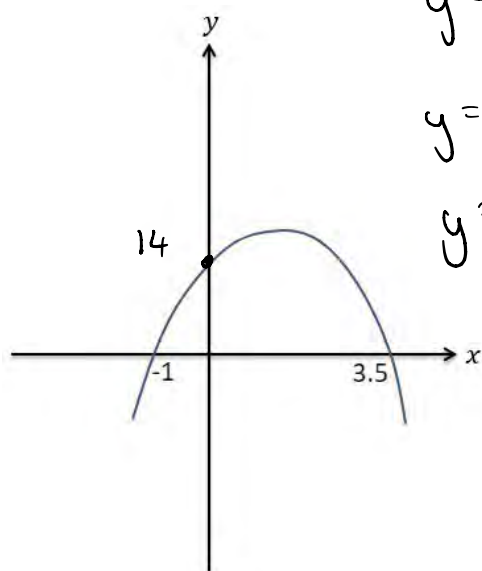
$$2(x^2 + x - 6)$$

$$y = 2x^2 + 2x - 12 \quad \checkmark$$

$$a = 2 \quad b = 2 \quad c = -12$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



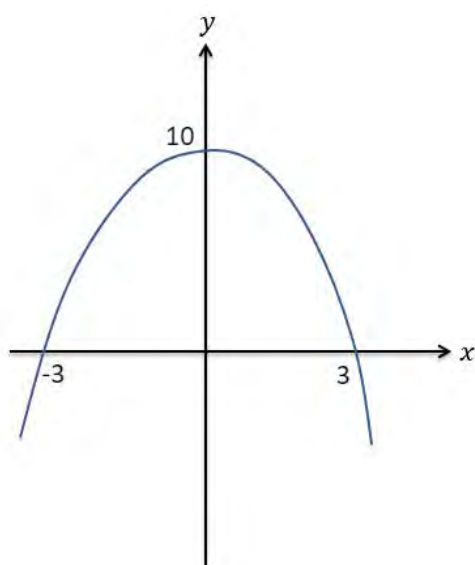
$$y = -4(x+1)(x-3.5)$$

$$y = -4(x^2 + x - 3.5x - 3.5)$$

$$y = -4(x^2 - 2.5x - 3.5)$$

$$\underline{\underline{y = -4x^2 + 10x + 14}}$$

Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



$$y = \frac{10}{-9}(x-3)(x+3)$$

$$y = \frac{10}{-9}(x^2 - 9)$$

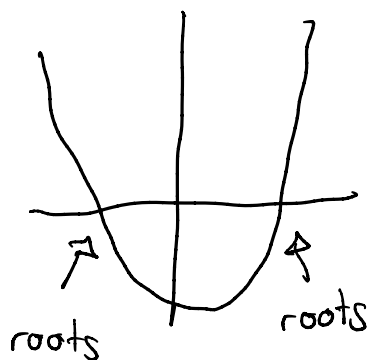
$$y = -\frac{10}{9}x^2 + 10$$

The Discriminant

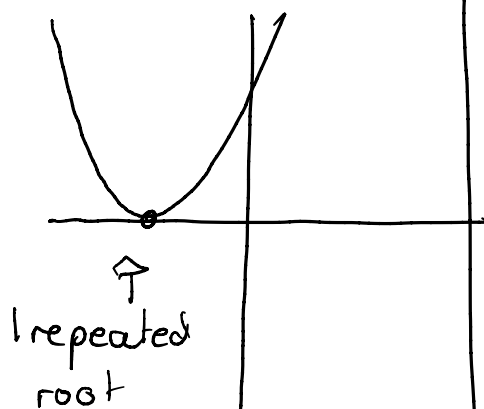
Quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant $\rightarrow b^2 - 4ac$

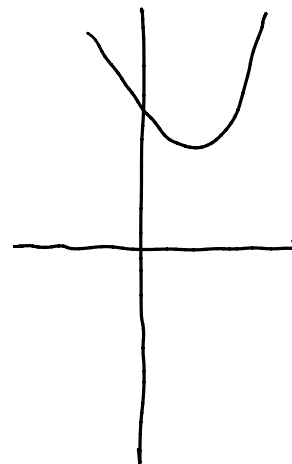
$b^2 - 4ac > 0$
 $\uparrow$
2 distinct roots



$b^2 - 4ac = 0$
 $\uparrow$
1 repeated root



$b^2 - 4ac < 0$
 $\uparrow$
no real roots



Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Quick fire questions:

Equation	Discriminant $b^2 - 4ac$	No. of distinct real roots
$x^2 + 3x + 4 = 0$	$(3)^2 - 4(1)(4) = -7$	no real roots
$x^2 - 4x + 1 = 0$	$(-4)^2 - 4(1)(1) = 12$	2 real roots
$x^2 - 4x + 4 = 0$	$(-4)^2 - 4(1)(4) = 0$	1 repeated root
$2x^2 - 6x - 3 = 0$	$(-6)^2 - 4(2)(-3) = 60$	2 real roots
$x - 4 - 3x^2 = 0$ $-3x^2 + x - 4 = 0$	$(1)^2 - 4(-3)(-4) = -47$	no real roots
$1 - x^2 = 0$ $-x^2 + 0x + 1 = 0$	$(0)^2 - 4(-1)(1) = 4$	2 real roots



Example:

8. The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(a) Find the value of p .

$$\uparrow$$

$$b^2 - 4ac = 0 \quad (4)$$

(b) For this value of p , solve the equation $x^2 + 2px + (3p + 4) = 0$.

(2)

Using $b^2 - 4ac = 0$

$$(2p)^2 - 4(1)(3p + 4) = 0$$

$$4p^2 - 12p - 16 = 0$$

$$p^2 - 3p - 4 = 0$$

$$(p - 4)(p + 1) = 0$$

$$\underline{\underline{p = 4}} \text{ or } \underline{\underline{p = -1}}$$

$\times$

$$x^2 + 8x + 16 = 0$$

$$(x + 4)^2 = 0$$

$$\underline{\underline{x = -4}} \text{ twice}$$

repeated root

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Test Your Understanding:

1. $x^2 + 5kx + (10k + 5) = 0$ where k is a positive constant.

Given that this equation has equal roots, determine the value of k .

$$\uparrow b^2 - 4ac = 0$$

Using $b^2 - 4ac = 0$

$$(5k)^2 - 4(1)(10k + 5) = 0$$

$$25k^2 - 40k - 20 = 0$$

$$5k^2 - 8k - 4 = 0$$

$$\underline{\underline{k = 2}} \quad k = -0.4$$

x

2. Find the range of values of k for which $x^2 + 6x + k = 0$ has two distinct real solutions.

$\uparrow$

$$(6)^2 - 4(1)(k) > 0$$

$$b^2 - 4ac > 0$$

$$36 - 4k > 0$$

$$36 > 4k$$

$$9 > k$$

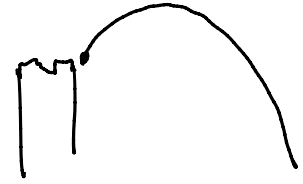
$$\underline{\underline{k < 9}}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Modelling

Example

A spear is thrown over level ground from the top of a tower.



The height, in metres, of the spear above the ground after t seconds is modelled by the function: $h(t) = 12.25 + 14.7t - 4.9t^2$, $t \geq 0$

- a) Interpret the meaning of the constant term 12.25 in the model.

12.25 is our initial height of our spear

- b) After how many seconds does the spear hit the ground?

$$h=0 \quad 0 = -4.9t^2 + 14.7t + 12.25$$

$$t = \underline{\underline{3.68}} \quad t = -0.679$$

- c) Write $h(t)$ in the form $A - B(t - C)^2$, where A , B and C are constants to be found.

$$\begin{aligned} h(t) &= -4.9t^2 + 14.7t + 12.25 \\ &= -4.9(t^2 - 3t) + 12.25 \\ &= -4.9\left[\left(t - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 12.25 \\ &= -4.9\left(t - \frac{3}{2}\right)^2 + \frac{441}{40} + 12.25 \\ &= \frac{931}{40} - 4.9\left(t - \frac{3}{2}\right)^2 \end{aligned}$$

- d) Using your answer to part c or otherwise, find the maximum height of the spear above the ground, and the time at which this maximum height is reached?

$$\left(\frac{3}{2}, \frac{931}{40}\right) \quad \text{max height} = 23.3\text{m}$$

$$\text{time} = 1.5\text{s}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Quadratics exam style question

A ball is thrown upwards from a rooftop 80m above the ground. It will reach a maximum vertical height and then fall back to the ground.

The height of the ball from ground at time t is h , given by the formula:

$$h = -16t^2 + 64t + 80$$

- a) Calculate the height reached by the ball after 1 second.
- b) Calculate the maximum height reached by the ball and after how many seconds from when it is thrown this maximum height is reached.
- c) Calculate how long will it take before the ball hits the ground.

Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.



Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Exam Questions

Q1.

The equation

$$(p - 1)x^2 + 4x + (p - 5) = 0, \text{ where } p \text{ is a constant}$$

has no real roots.

(a) Show that p satisfies $p^2 - 6p + 1 > 0$

(3)

(b) Hence find the set of possible values of p .

(4)

(Total for question = 7 marks)

Q2.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

(Total for question = 6 marks)

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Q3.

$$4x - 5 - x^2 = q - (x + p)^2$$

where p and q are integers.

(a) Find the value of p and the value of q .

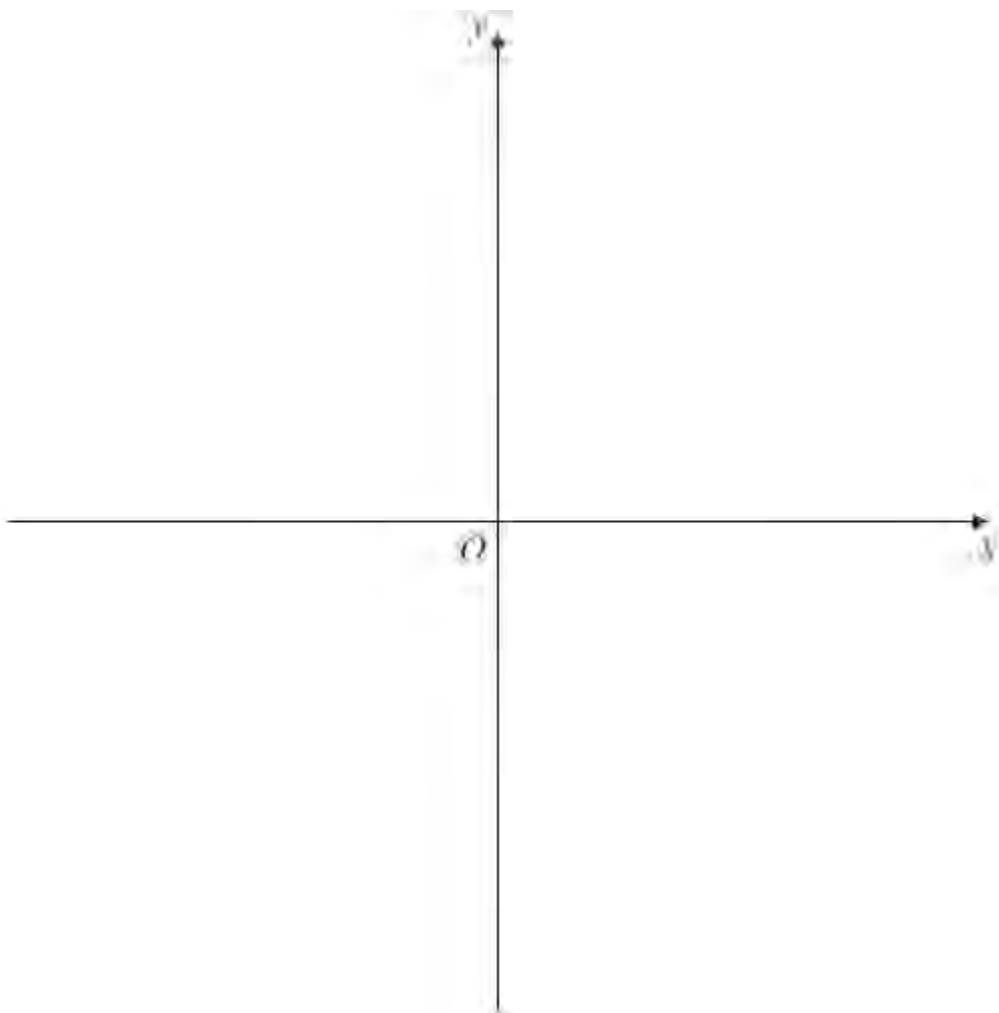
(3)

(b) Calculate the discriminant of $4x - 5 - x^2$

(2)

(c) On the axes on page 17, sketch the curve with equation $y = 4x - 5 - x^2$ showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)



(Total 8 marks)

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Q4.

$$f(x) = x^2 + (k + 3)x + k$$

where k is a real constant.

(a) Find the discriminant of $f(x)$ in terms of k .

(2)

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k , the equation $f(x) = 0$ has real roots.

(2)

(Total 6 marks)

Q5.

The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

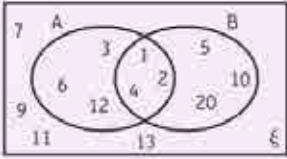
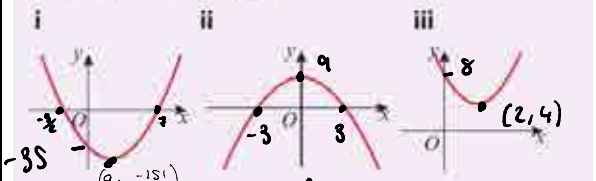
Find the value of p .

(4)

(Total 4 marks)

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Diagnostic for Chapter 3 Equations and Inequalities

1 $A = \{\text{factors of 12}\}$ $B = \{\text{factors of 20}\}$ Write down the numbers in each of these sets: a $A \cap B$ b $(A \cup B)'$  a) $A \cap B$ $\rightarrow 1, 2, 4$ b) $(A \cup B)'$ $\rightarrow 7, 9, 11, 13$	2 Simplify these expressions. a $\sqrt{75}$ b $\frac{2\sqrt{45} + 3\sqrt{32}}{6}$ a) $\sqrt{75} = \sqrt{25} \times \sqrt{3}$ $= 5\sqrt{3}$ b) $\frac{2(\sqrt{9}\sqrt{5}) + 3(\sqrt{16}\sqrt{2})}{6}$ $= \frac{2(3\sqrt{5}) + 3(4\sqrt{2})}{6}$ $= \frac{6\sqrt{5} + 12\sqrt{2}}{6}$ $= \sqrt{5} + 2\sqrt{2}$
3 Match the equations to the correct graph. Label the points of intersection with the axes and the coordinates of the turning point. a $y = 9 - x^2$ b $y = (x - 2)^2 + 4$ c $y = (x - 7)(2x + 5)$  a) $y = 9 - x^2$ $y = (3 - x)(3 + x)$ $0 = (3 - x)(3 + x)$ $x = 3 \quad x = -3$	b) $y = (x - 2)^2 + 4$ $TP \rightarrow (2, 4)$ $y = x^2 - 4x + 4 + 4$ $y = x^2 - 4x + 8$ c) $y = 2x^2 - 14x + 5x - 35$ $y = 2x^2 - 9x - 35$ $y = (x - 7)(2x + 5)$ $TP (9/2, -151/2)$ $x = 7 \quad x = -5/2$ $y = 2[x^2 - \frac{9}{2}x] - 35$ $y = 2[(x - \frac{9}{2})^2 - \frac{81}{4}] - 35$ $y = 2(x - \frac{9}{2})^2 - \frac{81}{2} - 35$ $y = 2(x - \frac{9}{2})^2 - \frac{151}{2}$

Simultaneous Equations

Simultaneous Equations Solution Sets

Scenario	Example	Solution Set
A single solution:	$\begin{aligned} x + y &= 9 \\ x - y &= 1 \end{aligned}$	$\begin{aligned} 2y &= 8 & 2x + 4 &= 9 \\ y &= 4 & 2x &= 5 \end{aligned} \quad (5, 4)$
Two solutions:	$\begin{aligned} \textcircled{1} \quad x^2 + y^2 &= 10 \\ x + y &= 4 \end{aligned}$	$\begin{aligned} x &= 4 - y \\ (4 - y)^2 + y^2 &= 10 \\ 16 - 8y + y^2 + y^2 &= 10 \\ 2y^2 - 8y + 6 &= 0 \\ y^2 - 4y + 3 &= 0 \\ (y - 3)(y - 1) &= 0 \\ y &= 3 & y &= 1 \\ x &= 1 & x &= 3 \end{aligned}$ $(1, 3) \quad (3, 1)$
No solutions:	$\begin{aligned} x + y &= 1 \\ x + y &= 3 \end{aligned}$	$0 = -2x$ impossible.
Infinitely large set of solutions:	$\begin{aligned} x + y &= 1 \\ 2x + 2y &= 2 \end{aligned}$	$y = -x + 1$ $x + y = 1$

Example (You can do this on your calculator!)

Solve the simultaneous equations

$$\textcircled{1} \quad 3x + y = 8$$

$$\textcircled{2} \quad 2x - 3y = 9$$

Method 1 : Elimination

$$\textcircled{1} \quad 3x + y = 8$$

$$\textcircled{2} \quad 2x - 3y = 9$$

$$\begin{aligned} \textcircled{1} \times 3 & \quad 9x + 3y = 24 \\ + & \quad 2x - 3y = 9 \end{aligned}$$

$$11x = 33$$

$$x = 3 \quad (3, -1)$$

$$\begin{aligned} \textcircled{1} \quad 3(3) + y &= 8 \\ y &= -1 \end{aligned}$$

Method 2: Substitution

$$\textcircled{1} \quad y = -3x + 8 \text{ sub into } \textcircled{2}$$

$$2x - 3(-3x + 8) = 9$$

$$2x + 9x - 24 = 9$$

$$11x = 33$$

$$x = 3$$

$$(3, -1)$$

$$\begin{aligned} \textcircled{1} \quad y &= -3(3) + 8 \\ y &= -1 \end{aligned}$$

Exercise 3A Page 40

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Linear and Quadratic

Example:

Solve the simultaneous equations:

$$(1) \quad x + 2y = 3$$

$$(2) \quad x^2 + 3xy = 10$$

$$(1) \quad x = 3 - 2y$$

sub (1) into (2)

$$(3 - 2y)^2 + 3(3 - 2y)y = 10$$

$$9 - 12y + 4y^2 + 9y - 6y^2 = 10$$

$$0 = 2y^2 + 3y + 1$$

$$0 = (2y + 1)(y + 1)$$

$$y = -\frac{1}{2}$$

$$\Rightarrow x = 3 - 2(-\frac{1}{2})$$

$$x = 4$$

$$(4, -\frac{1}{2})$$

$$x = 3 - 2(-1)$$

$$x = 5$$

$$(5, -1)$$

Test Your Understanding:

1. Solve the simultaneous equations: $3x^2 + y^2 = 21$ and $y = x + 1$

$$3x^2 + (x + 1)^2 = 21$$

$$3x^2 + x^2 + 2x + 1 = 21$$

$$4x^2 + 2x - 20 = 0$$

$$2x^2 + x - 10 = 0$$

$$(2x + 5)(x - 2) = 0$$

$$x = -\frac{5}{2} \quad x = 2$$

$$y = -\frac{5}{2} + 1 \quad (-\frac{5}{2}, -\frac{3}{2})$$

$$= -\frac{5}{2} + \frac{2}{2}$$

$$= -\frac{3}{2}$$

$$y = 2 + 1 \quad (2, 3)$$

$$y = 3$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Simultaneous Equations and Graphs

Examples:

1a. On the same axes, draw the graphs of $2x + y = 3$ and $y = x^2 - 3x + 1$

$$y = -2x + 3$$

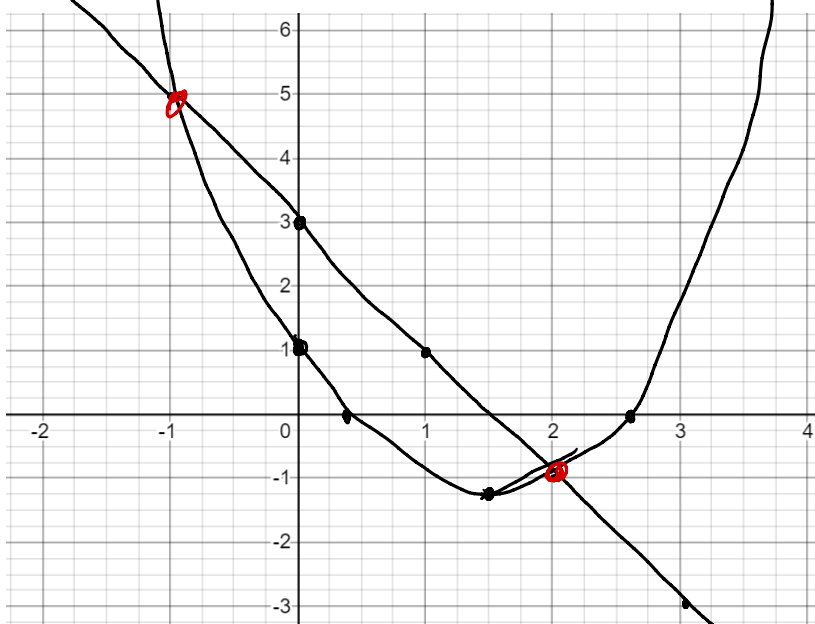
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$x = 2.62$$

$$\text{or } x = 0.382$$

$$y = -2x + 3$$

$$y = x^2 - 3x + 1$$



$$y = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 1$$

$$y = \left(x - \frac{3}{2}\right)^2 - \frac{5}{4}$$

$$\left(\frac{3}{2}, -\frac{5}{4}\right)$$

1b. Use your graph to write down the solutions to the simultaneous equations

$$(-1, 5) \quad (2, -1)$$

1c. What algebraic method could we have used to show the graphs would have intersected twice?

$$y = -2x + 3$$

$$y = x^2 - 3x + 1$$

$$-2x + 3 = x^2 - 3x + 1$$

$$0 = x^2 - x - 2$$

$$b^2 - 4ac = (-1)^2 - 4(1)(-2)$$

$$= 1 + 8$$

$$= 9$$

38

discriminant ≥ 0 i.e. 2 real solutions

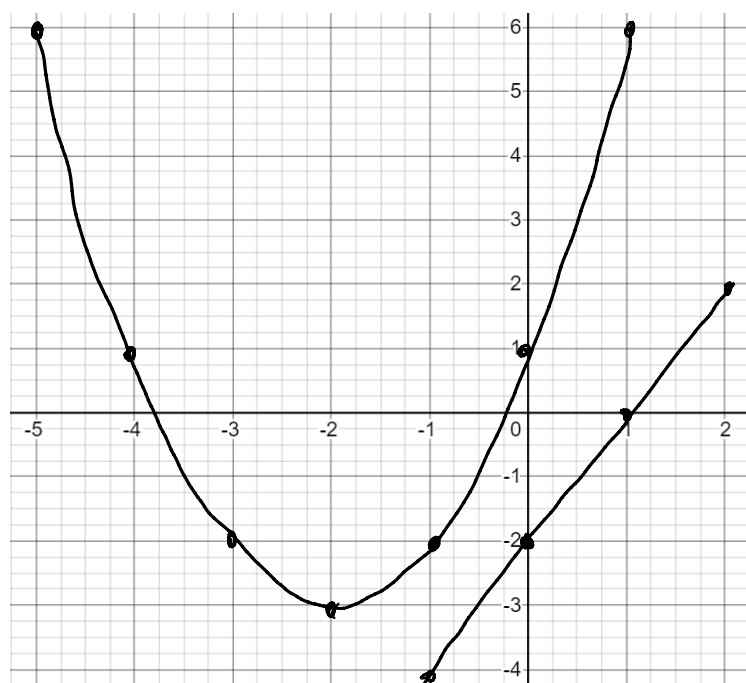
Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Example 2

a) On the same axes, draw the graphs of:

$$y = 2x - 2 \quad y = x^2 + 4x + 1$$

x	-5	-4	-3	-2	-1	0	1	2
y	6	1	-2	-3	-2	1	6	13



no' intersections

$$b^2 - 4ac < 0$$

b) Prove algebraically that the lines never meet

$$y = 2x - 2$$

$$y = x^2 + 4x + 1$$

$$2x - 2 = x^2 + 4x + 1$$

$$0 = x^2 + 2x + 3$$

$$\text{Discriminant} = (2)^2 - 4(1)(3)$$

$$= -8$$

< 0 ie no real solution.

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Test Your Understanding

The line with equation $y = 2x + 1$ meets the curve with equation $kx^2 + 2y + (k - 2) = 0$ at exactly one point. Given that k is a positive constant:

a) Find the value of k . $b^2 - 4ac = 0$

b) For this value of k , find the coordinates of this point of intersection

$$2y = -kx^2 - (k-2)$$

$$y = -\frac{k}{2}x^2 - \frac{(k-2)}{2}$$

a) $kx^2 + 2(2x + 1) + (k - 2) = 0$

$$kx^2 + 4x + 2 + k - 2 = 0$$

$$kx^2 + 4x + k = 0$$

$$b^2 - 4ac = 4^2 - 4(k)(k)$$

$$0 = 16 - 4k^2$$

$$4k^2 = 16$$

$$k^2 = 4$$

$$k = \pm 2$$

$$\underline{\underline{k = 2}}$$

b) $2x^2 + 2y = 0$

$$\textcircled{1} y = -x^2$$

$$\textcircled{2} y = 2x + 1$$

$$\textcircled{1} = \textcircled{2}$$

$$-x^2 = 2x + 1$$

$$0 = x^2 + 2x + 1$$

$$0 = (x + 1)^2$$

$$x = -1 \text{ repeated root.}$$

$$\textcircled{2} y = 2(-1) + 1$$

$$y = -1$$

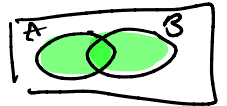
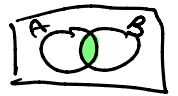
$$\underline{\underline{(-1, -1)}}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Set Builder Notation

Recap from GCSE:

- We use curly braces to list the values in a set, e.g. $A = \{1, 4, 6, 7\}$
- If A and B are sets then $A \cap B$ is the **intersection** of A and B , giving a set which has the elements in A **and** B .
- $A \cup B$ is the **union** of A and B , giving a set which has the elements in A **or** in B .
- $\emptyset$ is the empty set, i.e. the set with nothing in it. $\rightarrow 1, 2, 3, 4, \dots$
- Sets can also be infinitely large. $\mathbb{N}$ is the set of natural numbers (all positive integers), $\mathbb{Z}$ is the set of all integers (including negative numbers and 0) and $\mathbb{R}$ is the set of all real numbers (including all possible decimals).
- We write $x \in A$ to mean " x is a member of the set A ". So $x \in \mathbb{R}$



Quick Fire Examples

$$\{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$$

$$\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$$

$$\{1, 2\} \cap \{3, 4\} = \emptyset$$

Examples: $\rightarrow$ integers $\{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$

1. $\{2x : x \in \mathbb{Z}\} \rightarrow$ all even integers.
 $\{ \dots, -6, -4, -2, 0, 2, 4, 6, \dots \}$

$\rightarrow$ natural numbers $\{1, 2, 3, \dots\}$

2. $\{2^x : x \in \mathbb{N}\} = \{2, 4, 8, 16, 32, 64, \dots\}$

$\rightarrow \{2, 3, 5, 7, 11, 13, \dots\}$

3. $\{xy : x, y \text{ are prime}\} = \{4, 6, 9, 10, 15, 14, \dots\}$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Solving Inequalities

Linear inequalities Examples

1. $2x + 1 > 5$

$$2x \geq 4$$

$$x \geq 2$$



2. $3(x - 5) \geq 5 - 2(x - 8)$

$$3x - 15 \geq 5 - 2x + 16$$

$$5x \geq 36$$

$$x \geq 7.2$$

3. $-x \geq 2$

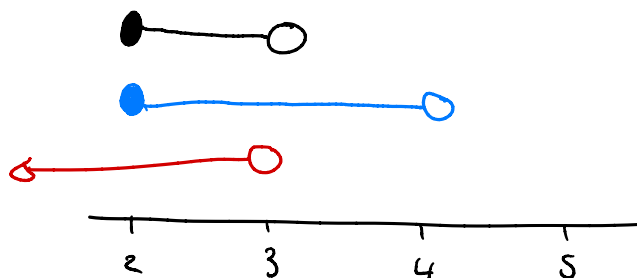
$$x \leq -2$$

Combining Inequalities

When combining inequalities always draw a number line to help!

Example:

If $x < 3$ and $2 \leq x < 4$, what is the combined solution set?



$$2 \leq x < 3$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Quadratic Inequalities:

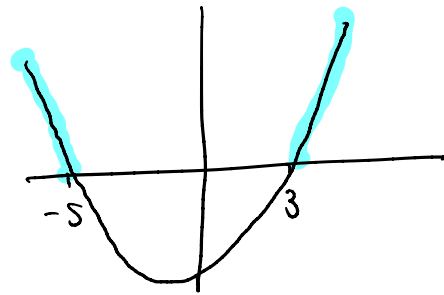
Examples

1. Solve $x^2 + 2x - 15 > 0$

$$0 = (x + 5)(x - 3)$$

$$x = -5 \quad x = 3$$

↖ above
x-axis

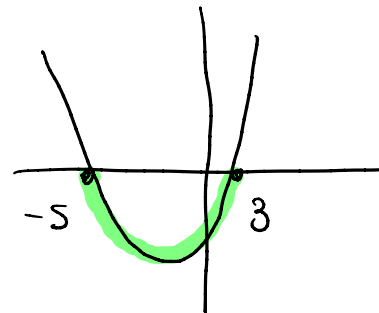


$$\{x : x < -5 \cup x > 3\} \quad x \in \mathbb{R}$$

2. Solve $x^2 + 2x - 15 \leq 0$

↖ below
x-axis

$$\{x : -5 \leq x \leq 3\}, \quad x \in \mathbb{R}$$

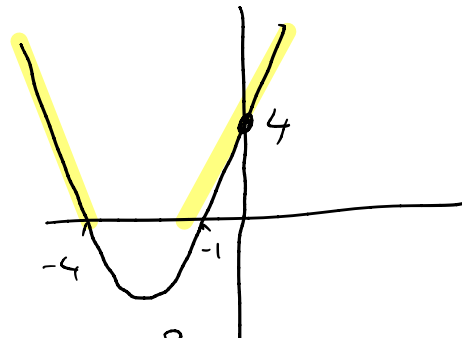


3. Solve $x^2 + 5x \geq -4$

$$x^2 + 5x + 4 \geq 0$$

$$(x + 4)(x + 1) = 0$$

$$x = -4 \quad x = -1$$



4. Solve $x^2 < 9$

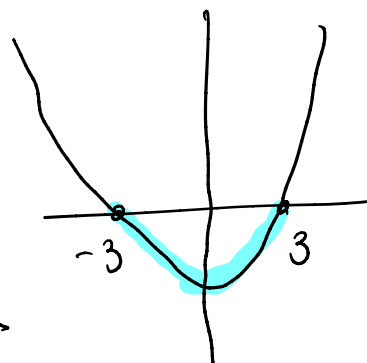
$$\{x : x < -3 \cup x > 3\} \quad x \in \mathbb{R}$$

$$x^2 - 9 < 0$$

$$(x + 3)(x - 3) = 0$$

$$x = 3, \quad x = -3$$

$$\{x : -3 < x < 3\} \quad x \in \mathbb{R}$$



Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Test Your Understanding

Find the set of values of x for which

(a) $3(x-2) < 8-2x$,

(2)

(b) $(2x-7)(1+x) < 0$,

(3)

(c) both $3(x-2) < 8-2x$ **and** $(2x-7)(1+x) < 0$.

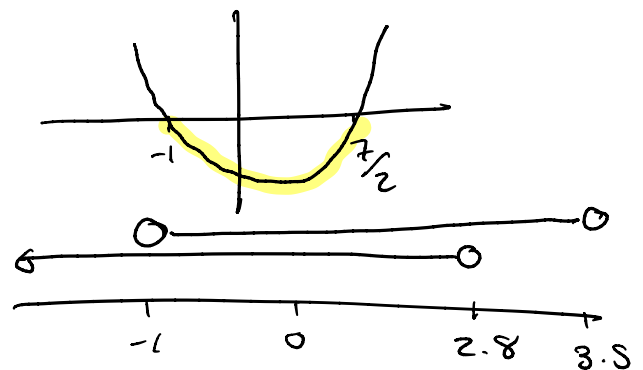
(1)

a) $3x - 6 < 8 - 2x$

$5x < 14$

$\{x: x < 2.8\} \quad x \in \mathbb{R}$ c)

b) $x = \frac{7}{2} \quad x = -1$



$\{x: -1 < x < 3.5\} \quad \{x: -1 < x < 2.8\} \quad x \in \mathbb{R}$

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

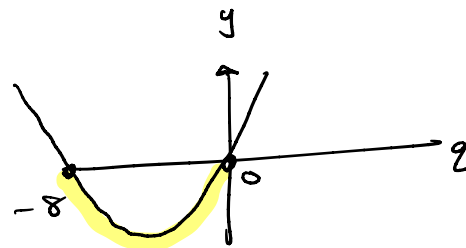
(2) $b^2 - 4ac < 0$

(b) Hence find the set of possible values of q .

(3)

a) $b^2 - 4ac \Rightarrow q^2 - 4(2q)(-1) < 0$
 $\Rightarrow q^2 + 8q < 0$

b) $q(q+8) = 0$
 $q = 0 \quad q = -8$



$\{q: -8 < q < 0\} \quad q \in \mathbb{R}$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Division by x

↪ We can't multiply x because it might be negative

Find the set of values for which $\frac{6}{x} > 2$, $x \neq 0$

↪ To get around this problem we multiply by x^2

$$\frac{6}{x} > 2$$

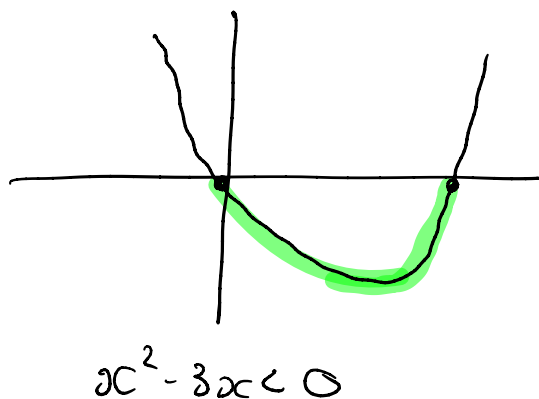
$$6x > 2x^2$$

$$0 > 2x^2 - 6x$$

$$0 > x^2 - 3x$$

$$0 = x(x - 3)$$

$$x = 0 \quad \text{or} \quad x = 3$$



$$\{x: 0 < x < 3\} x \in \mathbb{R}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Sketching Inequalities:

Examples

1. L_1 has equation $y = 12 + 4x$. L_2 has equation $y = x^2$.

The diagram shows a sketch of L_1 and L_2 on the same axes.

a) Find the coordinates of P_1 and P_2 , the points of intersection.

b) Hence write down the solution to the inequality

$$12 + 4x > x^2.$$

① = ②

$$12 + 4x = x^2$$

$$0 = x^2 - 4x - 12$$

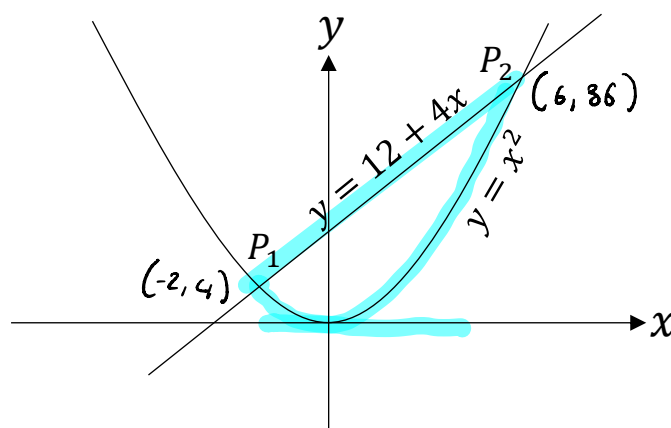
$$0 = (x - 6)(x + 2)$$

$$\underline{\underline{x = 6}}$$

$$\underline{\underline{x = -2}}$$

$$y = 36$$

$$y = 4$$



$$\{x: -2 < x < 6\} \quad x \in \mathbb{R}$$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

2. Shade the region that satisfies the inequalities:

① $2y + x < 14$

② $y \geq x^2 - 3x - 4$

① $y = -\frac{1}{2}x + 7$

$(0, 7)$

$y = 0 \quad 0 = -\frac{1}{2}x + 7$

$\frac{1}{2}x = 7$

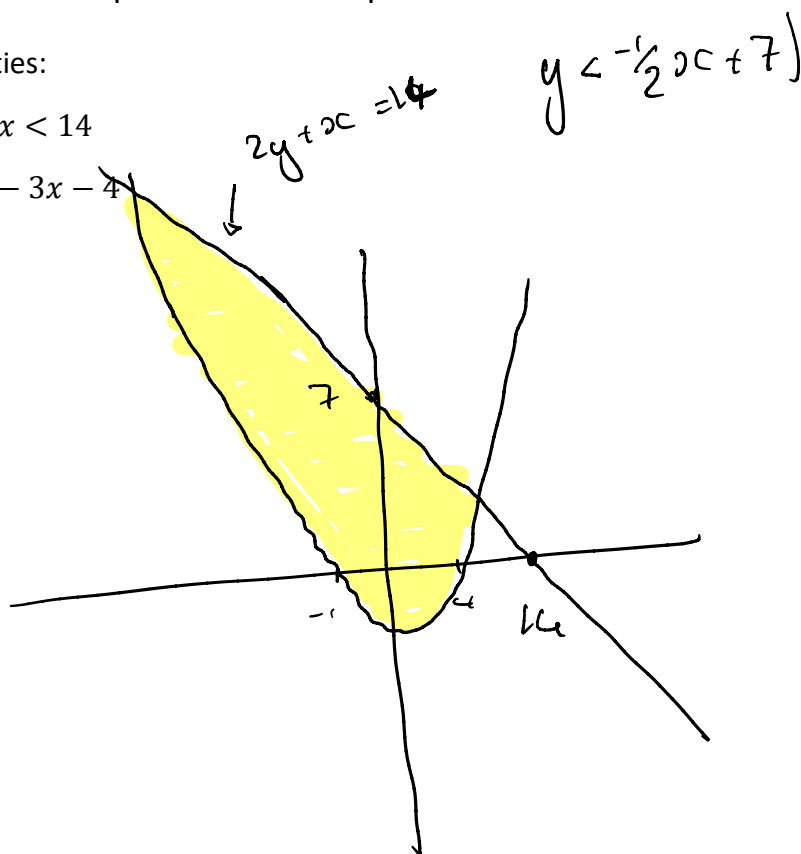
$x = 14$

$(14, 0)$

$y = x^2 - 3x - 4$

$y = (x - 4)(x + 1)$

$x = 4 \quad x = -1$



Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.



Exercise 3F/ 3G Page 53 – 55

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Exam Style Questions

Q1.

Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$

(2)

(b) $3x^2 + 8x - 3 < 0$

(4)

(Total 6 marks)

Q2.

Find the set of values of x for which

(a) $4x - 3 > 7 - x$

(2)

(b) $2x^2 - 5x - 12 < 0$

(4)

(c) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$

(1)

(Total 7 marks)

Q3.

The equation

$$(k + 3)x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x .

(a) Show that k satisfies

$$k^2 - 2k - 24$$

(4)

(b) Hence find the set of possible values of k .

(3)

(Total 7 marks)

Q4.

Find the set of values of x for which

(a) $3x - 7 > 3 - x$

(2)

(b) $x^2 - 9x \leq 36$

(4)

(c) **both** $3x - 7 > 3 - x$ **and** $x^2 - 9x \leq 36$

(1)

(Total 7 marks)

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

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Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

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Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

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