Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Instruction – Transition Task

- Aim to complete this booklet independently
- If you need support, use the video/ written solutions provided on the Urmston Grammar website.
- Complete all exam questions at the end of each section and mark them using the mark scheme provided.
- You do not need to do anything with the exercise boxes ->

Exercise 1A Page 3

First few lessons at Urmston Grammar

Lesson 1 – You will hand in your transition work to your teacher and then revise chapters 1, 2 and 3 in preparation for your skills test.
Lesson 2 – You will complete a skills test on chapters 1, 2, and 3
Lesson 3 – You will start new content.

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Diagnostic for Chapter 1 Algebraic Expressions

1 Simplify:

a $4m^2n + 5mn^2 - 2m^2n + mn^2 - 3mn^2$

b $3x^2 - 5x + 2 + 3x^2 - 7x - 12$

← GCSE Mathematics

2 Write as a single power of 2:

a $2^5 \times 2^3$ **b** $2^6 \div 2^2$

c $(2^3)^2$ ← GCSE Mathematics

a) $2m^2n + 3mn^2$

b) $6x^2 - 12x - 10$

2 a) 2^8

b) 2^4

c) 2^6

3 Expand:

a $3(x + 4)$ **b** $5(2 - 3x)$

c $6(2x - 5y)$ ← GCSE Mathematics

a) $3x + 12$

b) $10 - 15x$

c) $12x - 30y$

4 Write down the highest common factor of:

a 24 and 16 **b** $6x$ and $8x^2$

c $4xy^2$ and $3xy$ ← GCSE Mathematics

a)

1	24	1	16
2	12	2	8
3	8	4	4
4	6		

HCF = 8

b) $2x$

c) xy

5 Simplify:

a $\frac{10x}{5}$ **b** $\frac{20x}{2}$ **c** $\frac{40x}{24}$

a) $2x$

b) $10x$

c) $\frac{10}{6}x = \frac{5}{3}x$

Transition Task. Chapter 1 - Algebraic Expressions.
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Basic Index Laws

$$(a^n)^m = a^{n\times m} = a^{nm}$$

$$a^n \div a^m = a^{n-m}$$

$$a^n \times a^m = a^{n+m}$$

Examples

2. Simplify $(a^3)^2 \times 2a^2$

$$= a^6 \times 2a^2$$

$$= 2a^8$$

2. Simplify $(4x^3y)^3$ $= 4^3(x^3)^3y^3$

$$= 64x^9y^3$$

Alt

$$4x^3y \times 4x^3y \times 4x^3y$$

$$64x^9y^3$$

3. Simplify $2x^2(3+5x) - x(4-x^2)$

$$6x^2 + 10x^3 - 4x + x^3$$

$$= 11x^3 + 6x^2 - 4x$$

4. Simplify $\frac{x^3-2x}{3x^2}$ (2 methods)

$$\frac{\cancel{x}(x^2-2)}{3x^{\cancel{2}}} = \frac{x^2-2}{3x}$$

Alt

$$\frac{x^3}{3x^2} - \frac{2x}{3x^2}.$$

$$\frac{x}{3} - \frac{2}{3x}$$

Transition Task. Chapter 1 - Algebraic Expressions.
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Test Your Understanding:

1. Simplify $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$ $= \frac{4a^{10}}{a^4} \times 3a$

$= \frac{12a^{11}}{a^4}$

$= 12a^7$

2. Simplify $\frac{2x+x^5}{4x^3}$

$= \frac{x(2+x^4)}{4x^3}$

$= \frac{2+x^4}{4x^2}$

3.Expand and simplify $2x(3-x^2)-4x^3(3-x)$

$6x - 2x^3 - 12x^3 + 4x^4$

$= 4x^4 - 14x^3 + 6x$

4.Simplify $2^x \times 3^x$

$2 \times 2 \times 2 \ldots\ldots \times 2 \times 3 \times 3 \times 3 \ldots\ldots \times 3$

$= 2 \times 3 \times 2 \times 3 \times 2 \times 3 \ldots\ldots \times 2 \times 3$

$= 6^x$

Exercise 1A Page 3

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Negative and Fractional Indices

$a^{-1} = \frac{1}{a}$ $a^{1/2} = \sqrt{a}$ $a^{b/c} = \sqrt[c]{a^b}$

$a^{-b} = \frac{1}{a^b}$ $a^{1/b} = \sqrt[b]{a}$

1. Prove that $x^{\frac{1}{2}} = \sqrt{x}$

$x^{1/2} \times x^{1/2} = x^1$

$x^{1/2} = \sqrt{x}$

2. Evaluate $27^{-\frac{1}{3}}$

$27^{-1/3} = \left(\frac{1}{27}\right)^{1/3}$

$= \sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}}$

$= \frac{1}{3}$

3. Evaluate $32^{\frac{2}{5}}$

$\sqrt[5]{32}^{\,2} = 2^2$

$= 4$

4. Simplify $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

$\sqrt{\frac{1}{9}x^6y} = \frac{1}{3}x^3y^{1/2}$

2. Evaluate $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

$= \left(\frac{8}{27}\right)^{2/3}$

$= \left(\frac{2}{3}\right)^2$

$\frac{4}{9}$

6. If $b = \frac{1}{9}a^2$, determine $3b^{-2}$ in the form ka^n where k, n are constants

$3\left(\frac{1}{9}a^2\right)^{-2}$

$= 3\left(\frac{a^2}{9}\right)^{-2}$

$= 3\left(\frac{9}{a^2}\right)^2$

$= 3\left(\frac{81}{a^4}\right)$

$= \frac{243}{a^4}$

$= 243a^{-4}$

Exercise 1D Page 11

Transition Task. Chapter 1 - Algebraic Expressions.
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Brackets: Expanding

Example: $(x+1)(x+2)(x+3)$

$(x^2 + 2x + x + 2)(x + 3)$

$(x^2 + 3x + 2)(x + 3)$

$x^3 + 3x^2 + 2x + 3x^2 + 9x + 6$

$x^3 + 6x^2 + 11x + 6$

Questions

2. Expand and simplify

$$(x+5)(x-2)(x+1)$$

$(x^2 + 3x - 10)(x + 1)$

$x^3 + 3x^2 - 10x + x^2 + 3x - 10$

$x^3 + 4x^2 - 7x - 10$

2. Expand and simplify:

$$2(x-3)(x-4)$$

$2(x^2 - 7x + 12)$

$2x^2 - 14x + 24$

2. Expand and simplify:

$$(2x-1)^3$$

$(4x^2 - 4x + 1)(2x - 1)$

$8x^3 - 8x^2 + 2x - 4x^2 + 4x - 1$

$8x^3 - 12x^2 + 6x - 1$

Exercise 1B Page 5

Brackets: Factorising

Examples:

$+ \ -5$

$\times \ -14$

A M

1. $x^2 - 5x - 14$

$(x-7)(x+2)$

$\times \ -24 \quad +8$

$+ \ 5 \quad -3$

M A M

2. $2x^2 + 5x - 12$

$(2x-3)(x+4)$

	x	$+4$
$2x$	$2x^2$	$8x$
-3	$-3x$	-12

3. $4x^2 - 9$

$(2x+3)(2x-3)$

4. $x^3 - x$

$x(x^2-1)$

$x(x-1)(x+1)$

2. $x^3 + 3x^2 + 2x$

$x(x^2+3x+2)$

$x(x+2)(x+1)$

Test your understanding:

Factorise completely

1. $6x^2 + x - 2$

$(3x+2)(2x-1)$

2. $x^3 - 7x^2 + 12x$

$x(x^2-7x+12)$

$x(x-3)(x+4)$.

3. $x^4 - 1$

$(x^2-1)(x^2+1)$

$(x-1)(x+1)(x^2+1)$

4. $x^3 - 1$

$(x-1)(x^2+x+1)$.

at this stage we do not need to know this.

Exercise 1C Page 8

Transition Task. Chapter 1 - Algebraic Expressions.
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Surds:

Recap:

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ $\qquad \dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

$2 \times \sqrt{a} = 2\sqrt{a}$ $\qquad \sqrt{a}^2 = a$

Simplify:

1. $\sqrt{3} \times 2 \quad = 2\sqrt{3}$

2. $3\sqrt{5} \times 2\sqrt{5}$ $\quad 6 \times 5$

$= 30$

3. $\sqrt{8} \quad = \sqrt{4} \times \sqrt{2}$

$= 2\sqrt{2}$

4. $\sqrt{12} + \sqrt{27} \quad = \sqrt{4} \times \sqrt{3} + \sqrt{9} \times \sqrt{3}$

$= 2 \times \sqrt{3} + 3 \times \sqrt{3}$

$= 5\sqrt{3}$

2. $(\sqrt{8} + 1)(\sqrt{2} - 3)$

$= \sqrt{16} + \sqrt{2} - 3\sqrt{8} - 3$

$= 4 + \sqrt{2} - 3(2\sqrt{2}) - 3$

$= 1 + \sqrt{2} - 6\sqrt{2}$

$= 1 - 5\sqrt{2}$

Exercise 1E Page 11

Transition Task. Chapter 1 - Algebraic Expressions.
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Rationalising the denominator:

Examples:

1. $\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

2. $\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

3. $\frac{7}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{7\sqrt{7}}{7} = \sqrt{7}$

4. $\frac{15}{\sqrt{5}} + \sqrt{5} = \frac{15}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} + \sqrt{5} = \frac{15\sqrt{5}}{5} + \sqrt{5}$

$3\sqrt{5} + \sqrt{5}$

$= 4\sqrt{5}$

Test your understanding:

$\frac{12}{\sqrt{3}} = \frac{12 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$

$\frac{2}{\sqrt{6}} = \frac{2 \times \sqrt{6}}{\sqrt{6}\ \sqrt{6}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$

$\frac{4\sqrt{2}}{\sqrt{8}} = \frac{4\sqrt{2}}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{4\sqrt{16}}{8} = \frac{4 \times 4}{8} = \frac{16}{8} = 2$

Alt $\frac{\overset{2}{\cancel{4}}\cancel{\sqrt{2}}}{\underset{1}{\cancel{2}}\cancel{\sqrt{2}}} = 2$

More Complicated Examples:

1. $\frac{1}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \frac{\sqrt{2}-1}{2+\sqrt{2}-\sqrt{2}-1}$

$= \frac{\sqrt{2}-1}{1}$

$= \underline{\underline{\sqrt{2}-1}}$

2. $\frac{3}{(\sqrt{6}-2)} \times \frac{(\sqrt{6}+2)}{(\sqrt{6}+2)}$

$= \frac{3\sqrt{6}+6}{6-4}$

$= \frac{3\sqrt{6}+6}{2} \rightarrow \frac{3\sqrt{6}}{2} + 3$

3. $\frac{4}{\sqrt{3}+1} = \frac{4(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{4\sqrt{3}-4}{3-1}$

$= \frac{4\sqrt{3}}{2} - \frac{4}{2}$

$= \underline{\underline{2\sqrt{3}-2}}$

4. $\frac{3\sqrt{2}+4}{5\sqrt{2}-7} = \frac{(3\sqrt{2}+4)(5\sqrt{2}+7)}{(5\sqrt{2}-7)(5\sqrt{2}+7)}$

$= \frac{15(2) + 20\sqrt{2} + 21\sqrt{2} + 28}{25(2) - 49}$

$= \frac{30 + 20\sqrt{2} + 21\sqrt{2} + 28}{1}$

$= 58 + 41\sqrt{2}$

Test Your Understanding:

Rationalise the denominator and simplify

2. $\frac{4}{\sqrt{5}-2}$

$$\frac{4}{\sqrt{5}-2} \times \frac{(\sqrt{5}+2)}{(\sqrt{5}+2)} = \frac{4\sqrt{5}+8}{5-4}$$

$$= 4\sqrt{5}+8$$

2. $\frac{2\sqrt{3}-1}{3\sqrt{3}+1}$

$$\frac{(2\sqrt{3}-1)(3\sqrt{3}-1)}{(3\sqrt{3}+1)(3\sqrt{3}-1)}$$

$$= \frac{6(3)-3\sqrt{3}-2\sqrt{3}+1}{9(3)-1}$$

$$= \frac{18-5\sqrt{3}+1}{26}$$

$$\frac{19-5\sqrt{3}}{26}.$$

2. Solve $y(\sqrt{3}-1)=8$
Give your answer in the form $a+b\sqrt{3}$ where a and b are integers.

$$y = \frac{8}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$y = \frac{8\sqrt{3}+8}{3-1}$$

$$y = \frac{8\sqrt{3}+8}{2}$$

$$y = 4\sqrt{3}+4$$

Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.

Exercise 1F Page 15

Exam Questions – For revision purposes

Q1.

a) Simplify

$\sqrt{32} + \sqrt{18}$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

(2)

(b) Simplify

$$\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}$$

giving your answer in the form $b\sqrt{2} + c$, where b and c are integers.

(4)

Total 6 marks

Q2.

(a) Find the value of $16^{\frac{1}{4}}$

(2)

(b) Simplify $x(2x^{-\frac{1}{4}})^4$

(2)

(Total 4 marks)

Q3.

Simplify

$$\frac{5-2\sqrt{3}}{\sqrt{3}-1}$$

giving your answer in the form $p + q\sqrt{3}$, where p and q are rational numbers.

(4)

(Total 4 marks)

Q4.

(a) Evaluate $(32)^{\frac{3}{5}}$, giving your answer as an integer.

(2)

(b) Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$

(2)
(Total 4 marks)

Q5.

(a) Expand and simplify $(7 + \sqrt{5})(3 - \sqrt{5})$

(3)

(b) Express $\frac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

(3)
(Total 6 marks)

Q6.

Simplify

(a) $(3\sqrt{7})^2$

(1)

(b) $(8 + \sqrt{5})(2 - \sqrt{5})$

(3)
(Total 4 marks)

Q7.

Given that $32\sqrt{2} = 2^a$, find the value of a.

(3)
(Total 3 marks)

Q8.

(2) Express

$$(5 - \sqrt{8})(1 + \sqrt{2})$$

in the form $a + b\sqrt{2}$, where a and b are integers.

(3)

(ii) Express

$$\sqrt{80} + \frac{30}{\sqrt{5}}$$

in the form $c\sqrt{5}$, where c is an integer.

(3)
(Total 6 marks)

Q9.

Express 8^{2x+3} in the form 2^y, stating y in terms of x.

(2)
(Total 2 marks)

Transition Task. Chapter 1 - Algebraic Expressions. Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Diagnostic for Chapter 2 Quadratics

1 Solve the following equations:

a $3x + 6 = x - 4$

b $5(x + 3) = 6(2x - 1)$

c $4x^2 = 100$

d $(x - 8)^2 = 64$ ← GCSE Mathematics

a) $3x + 6 = x - 4$

$2x = -10$

$x = -5$

b) $5x + 15 = 12x - 6$

$21 = 7x$

$3 = x$

c) $4x^2 = 100$

$x^2 = 25$

$x = \pm 5$

Alt

$2x = \pm 10$

$x = \pm 5$

d) $(x-8)^2 = 64$

$x - 8 = \pm 8$

$x - 8 = 8$ $\quad x - 8 = -8$

$x = 16$ $\quad x = 0$

2 Factorise the following expressions:

a $x^2 + 8x + 15$

b $x^2 + 3x - 10$

c $3x^2 - 14x - 5$

d $x^2 - 400$

a) $(x+8)(x+5)$

b) $(x+5)(x-2)$

c) ⊗ -15 ? -15

⊕ -14 $+1$

	x	-5
$3x$	$3x^2$	$-15x$
1	x	-5

$(3x+1)(x-5)$

d) $x^2 - 400$

$(x-20)(x+20)$

3 Sketch the graphs of the following equations, labelling the points where each graph crosses the axes:

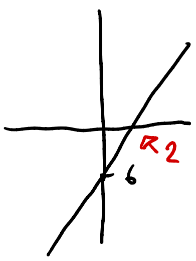
a $y = 3x - 6$

b $y = 10 - 2x$

c $x + 2y = 18$

d $y = x^2$ ← GCSE Mathematics

a)

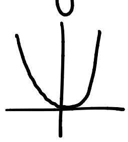


$0 = 3x - 6$

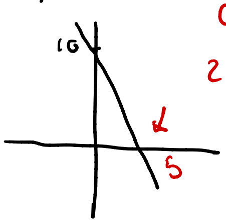
$6 = 3x$

$2 = x$

d) $y = x^2$



b)



$0 = 10 - 2x$

$2x = 10$

$x = 5$

c) $x + 2y = 18$

$\Rightarrow 2y = -x + 18$

$y = -\frac{1}{2}x + 9$

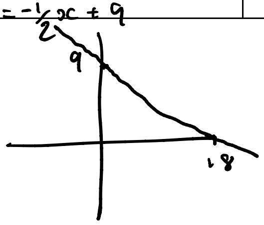
$x = 0$ $\quad 0 + 2y = 18$

$(0, 9)$ $\quad 2y = 18$

$y = 9$

$y = 0$ $\quad x = 18$

$(18, 0)$



4 Solve the following inequalities:

a $x + 8 < 11$

b $2x - 5 \geqslant 13$

c $4x - 7 \leqslant 2(x - 1)$

d $4 - x < 11$ ← GCSE Mathematics

When you divide or multiply by a negative interger. Remember to flip the sign.

a) $x + 8 < 11$

$x < 3$

b) $2x - 5 \geqslant 13$

$2x \geqslant 18$

$x \geqslant 9$

c) $4x - 7 \leq 2x - 2$

$2x \leq 5$

$x \leq 2.5$

d) $4 - x < 11$

$-x < 7$

$x\ 7 -7$

Alt

$4 < 11 + x$

$-7 < x$

$x > -7$

Transition Task. Chapter 1 - Algebraic Expressions.
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Solving Quadratic Equations

The 3 ways to solve a quadratic:
- → factorisation
- → formula
- → completing the square

Recap:

By factorisation

1. $x^2 + \overset{A}{5}x \overset{M}{-} 6 = 0$

$(x+6)(x-1) = 0$

$x+6=0$ or $x-1=0$

$x=-6$ $\qquad$ $x=1$

Using the Quadratic Formula

2. $x^2 + 5x - 6 = 0$

$a = 1$

$b = 5$

$c = -6$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-5 \pm \sqrt{5^2-4(1)(-6)}}{2(1)}$

$x = \dfrac{-5 \pm \sqrt{25+24}}{2}$

$x = \dfrac{-5 \pm \sqrt{49}}{2}$

$x = \dfrac{-5 \pm 7}{2}$

$\oplus$ $x = \dfrac{2}{2} = 1$

$\ominus$ $x = -\dfrac{12}{2}$

$= -6$

Examples

1. $(x-1)^2 = 5$

$x - 1 = \pm\sqrt{5}$

$x = 1 \pm \sqrt{5}$

2. Solve $x - 6\sqrt{x} + 8 = 0$

let $y = \sqrt{x}$

→ $y^2 = x$

$y^2 - 6y + 8 = 0$

$(y-4)(y-2) = 0$

$y = 4$ $\qquad$ $y = 2$

$\sqrt{x} = 4$ $\qquad$ $\sqrt{x} = 2$

$x = 16$ $\qquad$ $x = 4$

↳ $(\sqrt{x} - 4)(\sqrt{x} - 2) = 0$

$\sqrt{x} = 4$ $\qquad$ $\sqrt{x} = 2$

$x = 16$ $\qquad$ $x = 4$

Test your understanding

1. $(x+3)^2 = x+5$

$x^2 + 6x + 9 = x + 5$

$x^2 + 5x + 4 = 0$

$(x+4)(x+1) = 0$

$x = -4 \quad x = -1$

2. $(2x+1)^2 = 5$

$2x + 1 = \pm\sqrt{5}$

$2x = -1 \pm \sqrt{5}$

$x = \frac{-1 \pm \sqrt{5}}{2}$

3. $\sqrt{x+3} = x - 3$

$x + 3 = (x-3)^2$

$x + 3 = x^2 - 6x + 9$

$0 = x^2 - 7x + 6$

$0 = (x-6)(x-1)$

$x = 6 \quad x = 1$

4. $2x + \sqrt{x} - 1 = 0$

let $y = \sqrt{x}$

$y^2 = x$.

$2y^2 + y - 1 = 0$

$(2y-1)(y+1) = 0$

$-y$

$2y$

$2y = 1 \quad y = -1$

$y = \frac{1}{2} \quad \sqrt{x} = -1$

$\sqrt{x} = \frac{1}{2} \quad x = 1$

x

$x = \frac{1}{4}$

Exercise 2A/ 2B Page 20/ 22

Solving by Completing the Square

Completing the Square form:

$= d(x+a)^2 + b$

Worked Examples (a = 1): $ax^2 + bx + c$

1. $x^2 + 12x$

$(x+6)^2 - 36$

3. $x^2 - 2x$

$-(-1)^2$

$(x-1)^2 - 1$

2. $x^2 + 8x$

$(x+4)^2 - 16$

4. $x^2 - 6x + 7$

$(x-3)^2 - 9 + 7$

$(x-3)^2 - 2$

More complicated examples (a not equal to 1):

1. Express $2x^2 + 12x + 7$ in the form $a(x+b)^2 + c$

$$2[x^2 + 6x] + 7$$

$$2[(x+3)^2 - 9] + 7$$

$$2(x+3)^2 - 18 + 7$$

$$2(x+3)^2 - 11$$

2. Express $5 - 3x^2 + 6x$ in the form $a - b(x+c)^2$

$$-3x^2 + 6x + 5$$

$$-3[x^2 - 2x] + 5$$

$$-3[(x-1)^2 - 1] + 5$$

$$-3(x-1)^2 + 3 + 5$$

$$8 - 3(x-1)^2$$

Test Your Understanding:

1. Express $3x^2 - 18x + 4$ in the form $a(x+b)^2 + c$

$$3[x^2 - 6x] + 4$$

$$3[(x-3)^2 - 9] + 4$$

$$3(x-3)^2 - 27 + 4$$

$$3(x-3)^2 - 23$$

2. Express $20x - 5x^2 + 3$ in the form $a - b(x+c)^2$

$$-5x^2 + 20x + 3$$

$$-5[x^2 - 4x] + 3$$

$$-5[(x-2)^2 - 4] + 3$$

$$-5(x-2)^2 + 20 + 3$$

$$-5(x-2)^2 + 23$$

Solving by Completing the Square:

Note: Previously we factorised out the 3. This is because $3x^2 - 18x + 4$ on its own is an **expression,** so dividing by 3 (instead of factorising) would change the expression. However, in an equation, we can divide both sides by 3 without affecting the solutions.

Example

Solve the equation $3x^2 - 18x + 4 = 0$ by completing the square.

Completing the square

$$3(x^2 - 6x) + 4 = 0$$

$$3[(x-3)^2 - 9] + 4 = 0$$

$$3(x-3)^2 - 27 + 4 = 0$$

$$3(x-3)^2 - 23 = 0$$

Solving.

$$3(x-3)^2 = 23$$

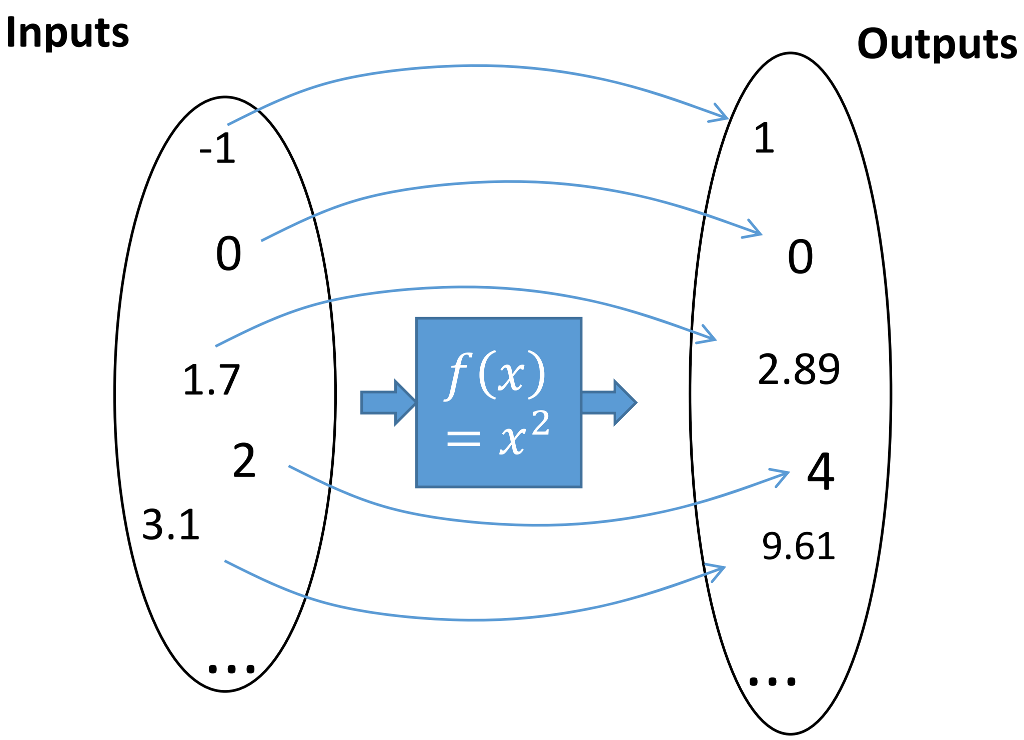
$$(x-3)^2 = \frac{23}{3}$$

$$x - 3 = \pm\sqrt{\frac{23}{3}}$$

$$x = 3 \pm \sqrt{\frac{23}{3}}$$

Exercise 2C/ 2D Page 23/ 24

Functions:



$f(x) = y$

Transition Task. Chapter 1 - Algebraic Expressions. Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Examples:

1. If $f(x) = x^2 - 3x$ and $g(x) = x + 5, \quad x \in \mathbb{R}$

   a) Find $f(-4)$

   b) Find the values of x for which $f(x) = g(x)$

   c) Find the roots of $f(x)$. → cross x-axis on your graph

   d) Find the roots of $g(x)$.

a) $f(-4) = (-4)^2 - 3(-4)$

$= 16 + 12$

$= \underline{28}$

b) $x^2 - 3x = x + 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5 \quad x = -1$

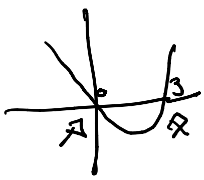
b) $f(x) = x^2 - 3x$

$f(x) = 0$

$0 = x^2 - 3x$

$0 = x(x-3)$

$x = 0 \quad x = 3$ → roots



d) $g(x) = x + 5$.

$g(x) = 0$.

$0 = x + 5$

$\underline{x = -5}$

2. Determine the minimum value of the function $f(x) = x^2 - 6x + 2$, and state the value of x for which this minimum occurs.

min $(x + a)^2 + b$

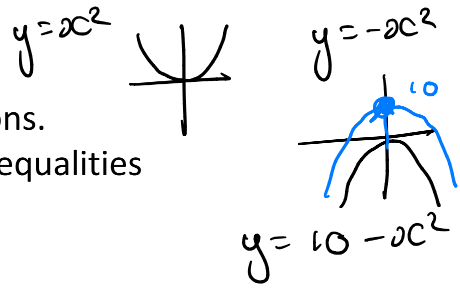
$(-a, b)$

$f(x) = x^2 - 6x + 2$

$= (x-3)^2 - 9 + 2$

$= (x-3)^2 - 7$

min $\underline{(3, -7)}$



Transition Task. Chapter 1 - Algebraic Expressions. Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Test Your Understanding:

$f(x)$	Completed square	Min/max value of $f(x)$	x for which this min/max occurs
x^2+4x+9	$(x+2)^2+5$	$(-2,5)$ min	-2
$x^2-10x+21$	$(x-5)^2-4$	$(5,-4)$ min	5
$10-x^2$	$-(x+0)^2+10$	$(0,10)$ max	0
$8-x^2+6x$ $-(x^2-6x)+8$	$-[(x-3)^2-9]+8$ $-(x-3)^2+9+8$ $-(x-3)^2+17$	$(3,17)$ max	3

1. Find the minimum value of $f(x)=2x^2+12x-5$ and state the value of x for which this occurs.

$$f(x)=2[x^2+6x]-5$$
$$=2[(x+3)^2-9]-5$$
$$=2(x+3)^2-18-5$$
$$=2(x+3)^2-23$$

min $(-3,-23)$

$x=-3$

2. Find the roots of the function $f(x)=2x^2+3x+1$

$$f(x)=(2x+1)(x+1)$$

$x=-\frac{1}{2}$ $\quad$ $x=-1$

3. Find the roots of the function $f(x)=x^4-x^2-6$

$y=x^2$ $\quad$ $\sqrt{y}=x$

$y^2=x^4$

$$f(\sqrt{y})=y^2-y-6$$
$$=(y-3)(y+2)$$

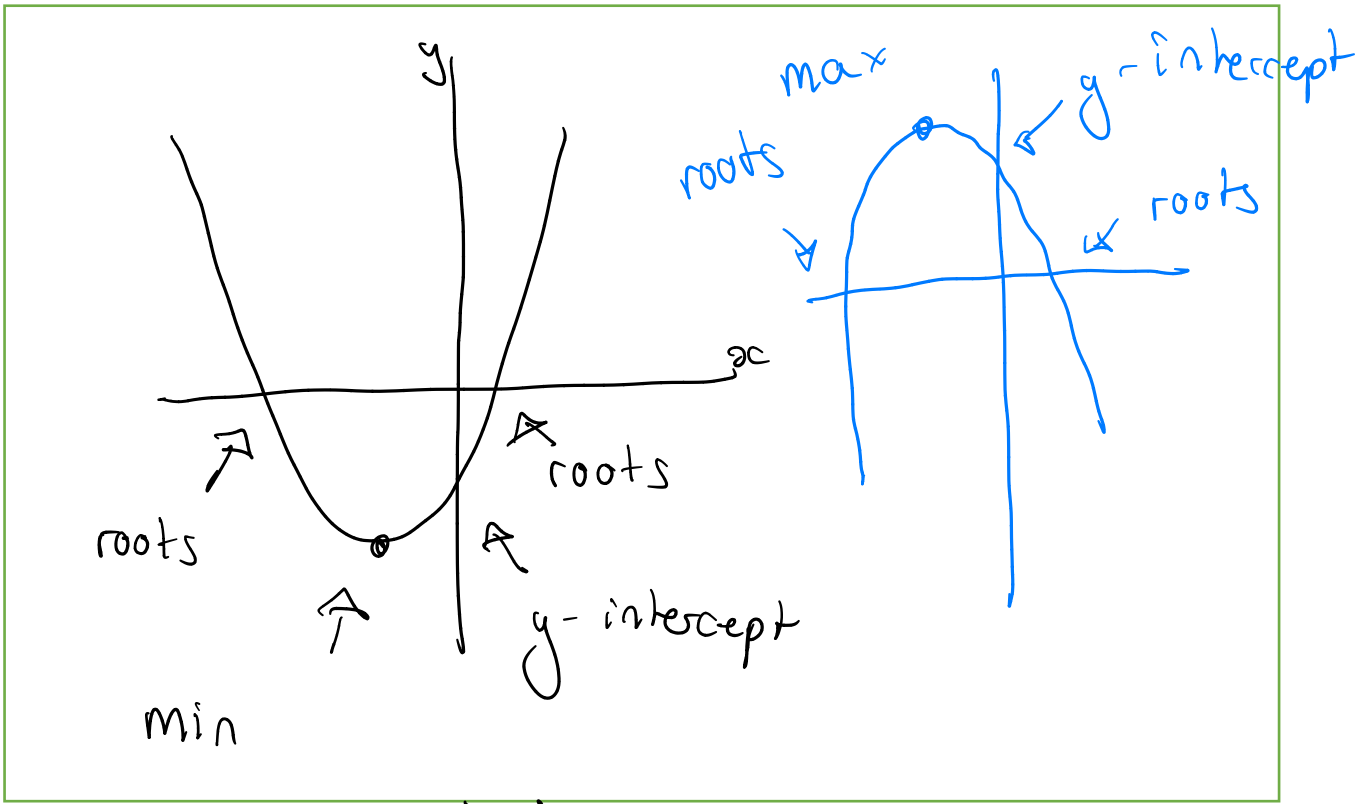
$y=3$ $\quad$ $y=-2$

$x^2=3$ $\quad$ $x^2=-2$ $\quad$ $\times$

$x=\pm\sqrt{3}$

Exercise 2E Page 27

Quadratic Graphs:



Example: Sketch the graph of $y = x^2 + 3x - 4$ and find the coordinates of the turning point.

$(0, -4)$

$$y = x^2 + 3x - 4$$

$$y = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 4$$

$$y = \left(x + \frac{3}{2}\right)^2 - \frac{25}{4}$$

min $\left(-\frac{3}{2}, -\frac{25}{4}\right)$

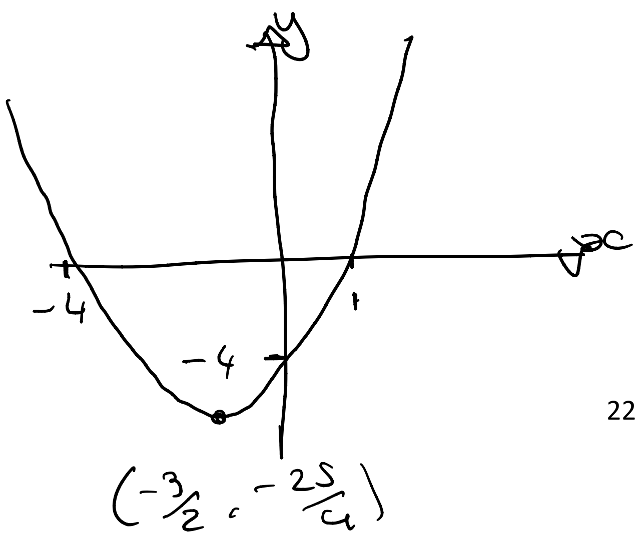
$$-\frac{9}{4} - \frac{16}{4}$$

$$-\frac{25}{4}$$

find roots $y = x^2 + 3x - 4$

$0 = (x+4)(x-1)$

$x = -4 \quad x = 1$



Example

∩ $(0, -3)$

Sketch the graph of $y = 4x - 2x^2 - 3$ and find the coordinates of the turning point. Write down the equation of the line of symmetry.

$y = -2x^2 + 4x - 3$

$y = -2[x^2 - 2x] - 3$

$y = -2[(x-1)^2 - 1] - 3$

$y = -2(x-1)^2 + 2 - 3$

$y = -2(x-1)^2 - 1$

max $(1, -1)$

Root

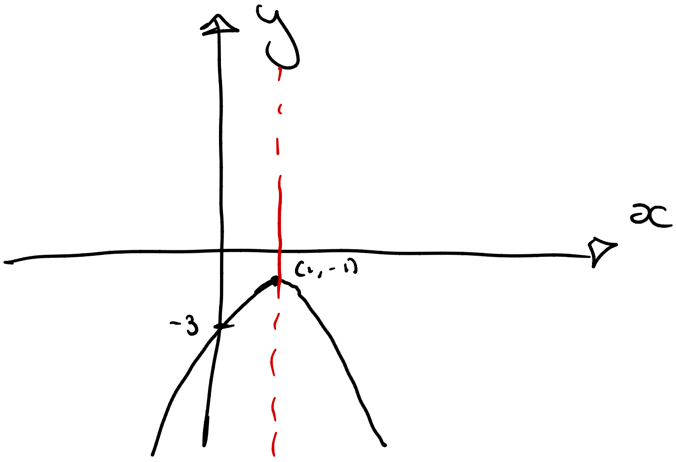
$0 = -2(x-1)^2 - 1$

$1 = -2(x-1)^2$

$\frac{1}{-2} = (x-1)^2$

$\sqrt{\frac{1}{-2}}$ = X error.

no roots

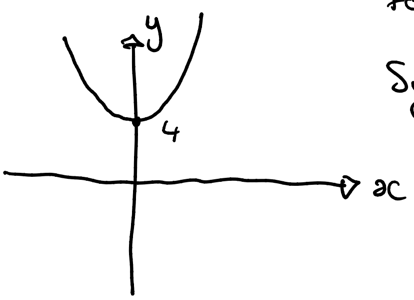


$x = 1$ → line of symmetry

Test Your Understanding

Sketch the following, indicating any intercepts with the axis, the turning point and the equation of the line of symmetry.

1. $y = x^2 + 4$



turning point = $(0, 4)$

Symmetry $x = 0$

2. $y = x^2 - 7x + 10$

$y = (x-5)(x-2)$

$x = 5 \qquad x = 2$

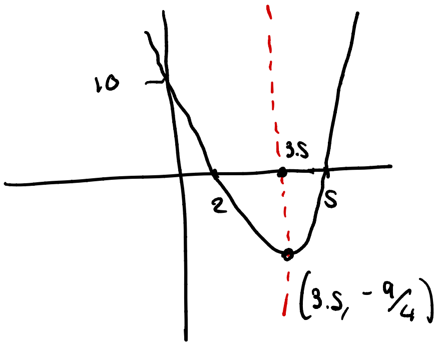
line of symmetry $\Rightarrow x = 3.5$

min point

$y = (3.5)^2 - 7(3.5) + 10$

$y = -\frac{9}{4}$

$\left(3.5, -\frac{9}{4}\right)$



$\frac{5+2}{2} = 3.5$

3. $y = 5x + 3 - 2x^2$

$y = -2x^2 + 5x + 3$

$y = -2\left[x^2 - \frac{5}{2}x\right] + 3$

$y = -2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] + 3$

$y = -2\left(x - \frac{5}{4}\right)^2 + \frac{25}{8} + 3$

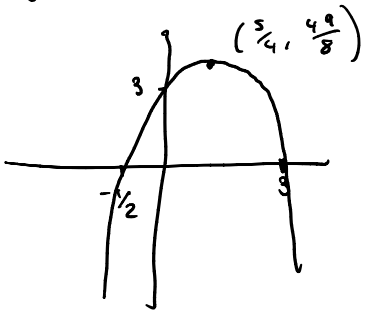
$y = -2\left(x - \frac{5}{4}\right)^2 + \frac{49}{8}$

$\left(+\frac{5}{4}, \frac{49}{8}\right)$ max

from calc $\rightarrow x = 3 \quad x = -\frac{1}{2}$

line of symmetry $x = +\frac{5}{4}$

y intercept $(3, 0)$

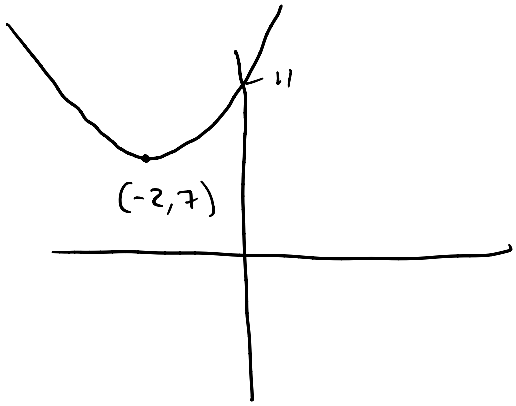


4. $y = x^2 + 4x + 11$

from calc. $x = -2 + \sqrt{7}\,i$ → complex number

no roots ?

$(-2, 7)$

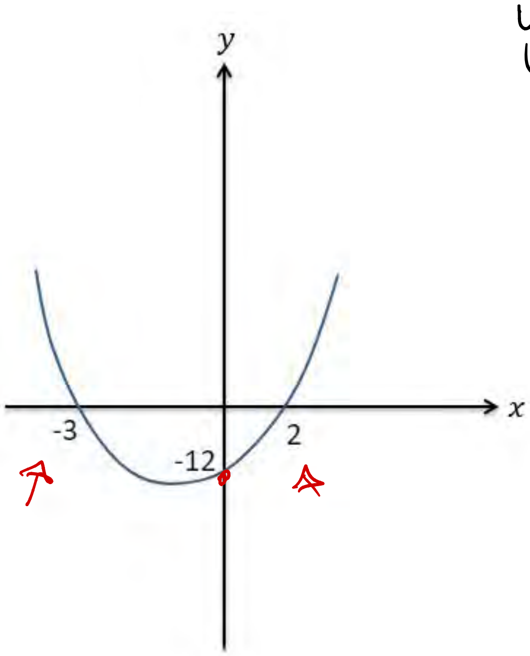


Complete.

$y = (x+2)^2 - 4 + 11$

$\underline{\underline{y = (x+2)^2 + 7}}$

Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



$y = 2(x+3)(x-2)$

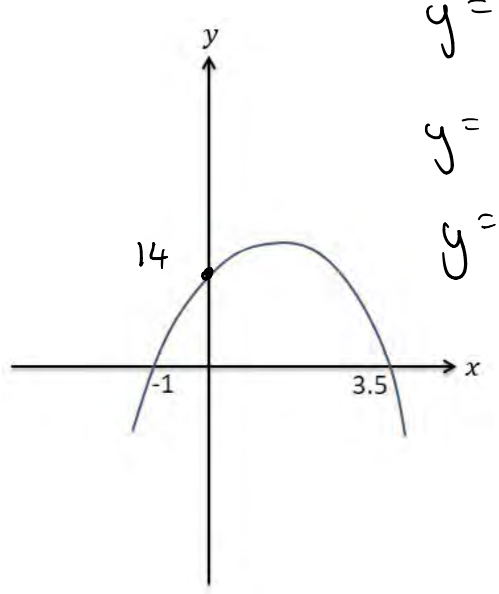
$x^2 + 3x - 2x - 6$

$2(x^2 + x - 6)$

$y = 2x^2 + 2x - 12$ ✓

$a = 2 \quad b = 2 \quad c = -12$

Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



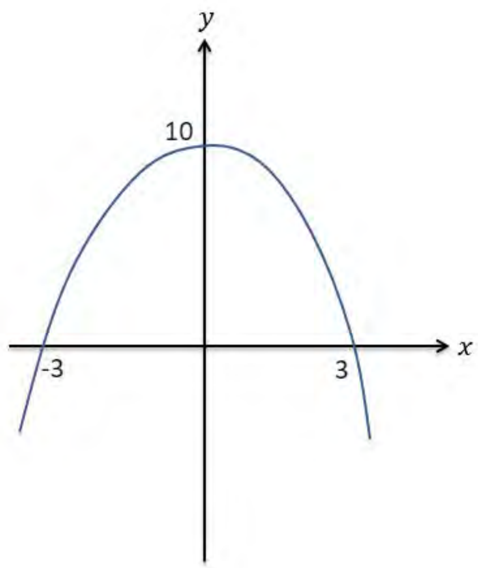
$y = -4(x+1)(x-3.5)$

$y = -4(x^2 + x - 3.5x - 3.5)$

$y = -4(x^2 - 2.5x - 3.5)$

$y = -4x^2 + 10x + 14$

Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



$y = \frac{10}{-9}(x-3)(x+3)$

$y = \frac{10}{-9}(x^2 - 9)$

$y = \frac{-10}{9}x^2 + 10$

Exercise 2F Page

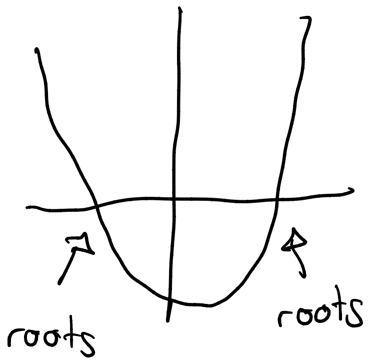
The Discriminant

Quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant $\rightarrow$ $b^2 - 4ac$

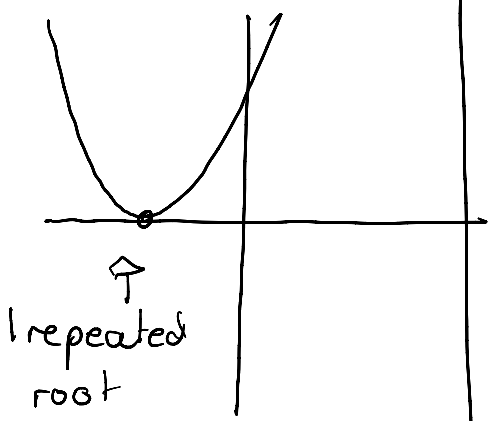
$b^2 - 4ac > 0$

2 distinct roots



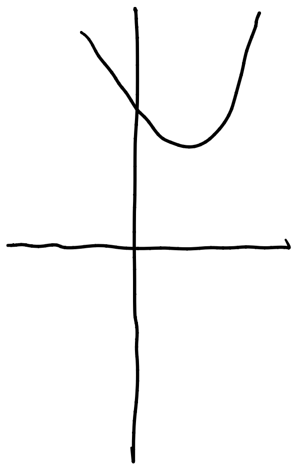
$b^2 - 4ac = 0$

1 repeated root



$b^2 - 4ac < 0$

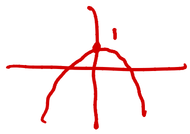
no real roots



Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Quick fire questions:

Equation	Discriminant b^2-4ac	No. of distinct real roots
$x^2+3x+4=0$	$(3)^2-4(1)(4)=-7$	no real roots
$x^2-4x+1=0$	$(-4)^2-4(1)(1)=12$	2 real roots
$x^2-4x+4=0$	$(-4)^2-4(1)(4)=0$	1 repeated root
$2x^2-6x-3=0$	$(-6)^2-4(2)(-3)=60$	2 real roots
$x-4-3x^2=0$ $-3x^2+x-4=0$	$(1)^2-4(-3)(-4)=-47$	no real roots
$1-x^2=0$ $-x^2+0x+1=0$	$(0)^2-4(-1)(1)=4$	2 real roots



Example:

a b c

8. The equation $x^2+2px+(3p+4)=0$, where p is a positive constant, has equal roots.

(*a*) Find the value of p. $b^2-4ac=0$ **(4)**

(*b*) For this value of p, solve the equation $x^2+2px+(3p+4)=0$. **(2)**

Using $b^2-4ac=0$

$(2p)^2-4(1)(3p+4)=0$

$4p^2-12p-16=0$

$p^2-3p-4=0$

$(p-4)(p+1)=0$

$p=4$ or $p=-1$ ×

$x^2+8x+16=0$

$(x+4)^2=0$

$x=-4$ twice

repeated root

Test Your Understanding:

1. $\overset{a}{1}x^2 + \overset{b}{5k}x + \overset{c}{(10k+5)} = 0$ where k is a positive constant.

Given that this equation has equal roots, determine the value of k.

$b^2 - 4ac = 0$

Using $b^2 - 4ac = 0$

$(5k)^2 - 4(1)(10k+5) = 0$

$25k^2 - 40k - 20 = 0$

$5k^2 - 8k - 4 = 0$

$\underline{\underline{k = 2}}$ $\quad k = -0.4$ ×

2. Find the range of values of k for which $x^2 + 6x + k = 0$ has two distinct real solutions.

$b^2 - 4ac > 0$

$(6)^2 - 4(1)(k) > 0$

$36 - 4k > 0$

$36 > 4k$

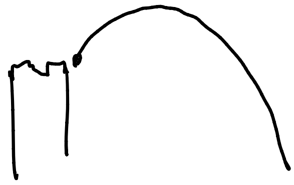
$9 > k$

$\underline{\underline{k < 9}}$

Exercise 2G Page 32

Modelling

Example



A spear is thrown over level ground from the top of a tower.

The height, in metres, of the spear above the ground after t seconds is modelled by the function: $h(t) = 12.25 + 14.7t - 4.9t^2, \quad t \geq 0$

a) Interpret the meaning of the constant term 12.25 in the model.

12.25 is our initial height of our spear

b) After how many seconds does the spear hit the ground?

$h = 0$ $\quad 0 = -4.9t^2 + 14.7t + 12.25$

$\underline{\underline{t = 3.68}}$ $\quad t = -0.679$ ✗

c) Write $h(t)$ in the form $A - B(t - C)^2$, where A, B and C are constants to be found.

$$h(t) = -4.9t^2 + 14.7t + 12.25$$
$$= -4.9(t^2 - 3t) + 12.25$$
$$= -4.9\left[\left(t - \frac{3}{2}\right)^2 - \frac{9}{4}\right] + 12.25$$
$$= -4.9\left(t - \frac{3}{2}\right)^2 + \frac{441}{40} + 12.25$$
$$= \frac{931}{40} - 4.9\left(t - \frac{3}{2}\right)^2$$

d) Using your answer to part c or otherwise, find the maximum height of the spear above the ground, and the time at which this maximum height is reached?

$\left(\frac{3}{2}, \frac{931}{40}\right)$ max height = 23.3m

time = 1.5s

Exercise 2H Page 34

Quadratics exam style question

A ball is thrown upwards from a rooftop 80m above the ground. It will reach a maximum vertical height and then fall back to the ground.

The height of the ball from ground at time t is h, given by the formula:

$$h = -16t^2 + 64t + 80$$

a) Calculate the height reached by the ball after 1 second.

b) Calculate the maximum height reached by the ball and after how many seconds from when it is thrown this maximum height is reached.

c) Calculate how long will it take before the ball hits the ground.

Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.

Exam Questions

Q1.

The equation

$$(p - 1)x^2 + 4x + (p - 5) = 0, \text{ where } p \text{ is a constant}$$

has no real roots.

(a) Show that p satisfies $p^2 - 6p + 1 > 0$

(3)

(b) Hence find the set of possible values of p.

(4)

(Total for question = 7 marks)

Q2.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

(Total for question = 6 marks)

Q3.

$$4x - 5 - x^2 = q - (x + p)^2$$

where p and q are integers.

(a) Find the value of p and the value of q.

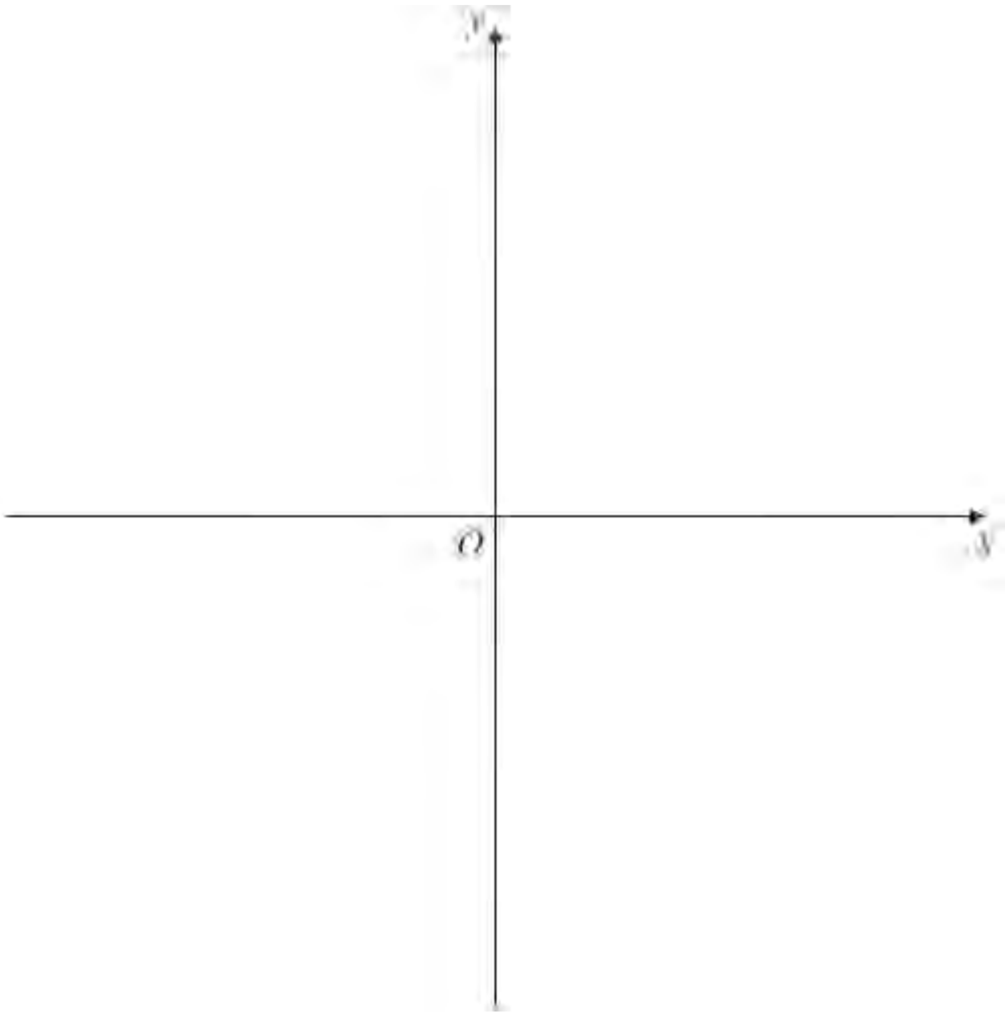
(3)

(b) Calculate the discriminant of $4x - 5 - x^2$

(2)

(c) On the axes on page 17, sketch the curve with equation $y = 4x - 5 - x^2$ showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)



(Total 8 marks)

Q4.

$$f(x) = x^2 + (k+3)x + k$$

where k is a real constant.

(a) Find the discriminant of f(x) in terms of k.

(2)

(b) Show that the discriminant of f(x) can be expressed in the form $(k + a)^2 + b$, where a and b are integers to be found.

(2)

(c) Show that, for all values of k, the equation f(x) = 0 has real roots.

(2)
(Total 6 marks)

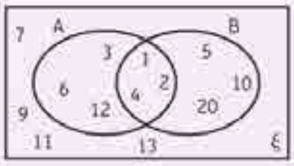
Q5.
The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(4)
(Total 4 marks)

Diagnostic for Chapter 3 Equations and Inequalities

1 A = {factors of 12}
B = {factors of 20}
Write down the numbers in each of these sets:
a $A \cap B$ **b** $(A \cup B)'$



a) $A \cap B$
$\rightarrow 1, 2, 4$

b) $(A \cup B)'$
$\rightarrow 7, 9, 11, 13$

2 Simplify these expressions.
a $\sqrt{75}$ **b** $\frac{2\sqrt{45} + 3\sqrt{32}}{6}$

a) $\sqrt{75} = \sqrt{25} \times \sqrt{3}$
$= 5\sqrt{3}$

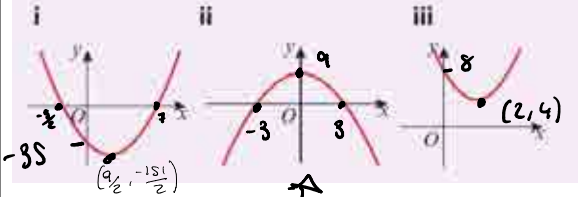
b) $\frac{2(\sqrt{9}\sqrt{5}) + 3(\sqrt{16}\sqrt{2})}{6}$

$= \frac{2(3\sqrt{5}) + 3(4\sqrt{2})}{6}$

$= \frac{6\sqrt{5} + 12\sqrt{2}}{6}$

$= \sqrt{5} + 2\sqrt{2}$

3 Match the equations to the correct graph. Label the points of intersection with the axes and the coordinates of the turning point.
a $y = 9 - x^2$ **b** $y = (x-2)^2 + 4$
c $y = (x-7)(2x+5)$



a) $y = 9 - x^2$
$y = (3 - x)(3 + x)$
$0 = (3 - x)(3 + x)$
$x = 3 \quad x = -3$

b) $y = (x-2)^2 + 4$
TP $\Rightarrow (2, 4)$

$y = x^2 - 4x + 4 + 4$
$y = x^2 - 4x + 8$

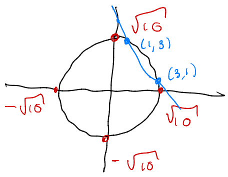
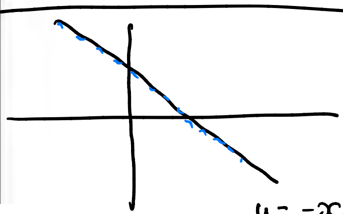
c) $y = 2x^2 - 14x + 5x - 35$
$y = 2x^2 - 9x - 35$

$y = (x-7)(2x+5)$ TP $\left(\frac{9}{2}, -\frac{151}{2}\right)$
$x = 7 \quad x = -\frac{5}{2}$

$y = 2\left[x^2 - \frac{9}{2}x\right] - 35$
$y = 2\left[\left(x - \frac{9}{2}\right)^2 - \frac{81}{4}\right] - 35$
$y = 2\left(x - \frac{9}{2}\right)^2 - \frac{81}{2} - 35$
$y = 2\left(x - \frac{9}{2}\right)^2 - \frac{151}{2}$

Simultaneous Equations

Simultaneous Equations Solution Sets

Scenario	Example	Solution Set
A single solution:	$x + y = 9$ $-$ $x - y = 1$	$2y = 8$, $y = 4$ $x + 4 = 9$, $x = 5$ $(5, 4)$
Two solutions:	$x^2 + y^2 = r^2$ ① $x^2 + y^2 = 10$ ② $x + y = 4$	$x = 4 - y$ ① $(4-y)^2 + y^2 = 10$ $16 - 8y + y^2 + y^2 = 10$ $2y^2 - 8y + 6 = 0$ $y^2 - 4y + 3 = 0$ $(y-3)(y-1) = 0$ $y = 3$, $x = 1$ $y = 1$, $x = 3$ $(1,3)$ $(3,1)$ 
No solutions:	$x + y = 1$ $-$ $x + y = 3$	$0 = -2$ × impossible.
Infinitely large set of solutions:	$x + y = 1$ $2x + 2y = 2$ ½	 $y = -x + 1$ $x + y = 1$

Example (You can do this on your calculator!)

Solve the simultaneous equations

$$① \; 3x + y = 8$$
$$② \; 2x - 3y = 9$$

Method 1 : Elimination

① $3x + y = 8$

② $2x - 3y = 9$

① × 3 $\quad 9x + 3y = 24$

$+ \quad 2x - 3y = 9$

$11x = 33$

$x = 3 \qquad (3, -1)$

① $3(3) + y = 8$

$y = -1$

Method 2: Substitution

① $y = -3x + 8$ sub into ②

$2x - 3(-3x + 8) = 9$

$2x + 9x - 24 = 9$

$11x = 33$

$x = 3$

① $y = -3(3) + 8$

$y = -1$

$(3, -1)$

Exercise 3A Page 40

Linear and Quadratic

Example:

Solve the simultaneous equations:

$$① \quad x + 2y = 3$$
$$② \quad x^2 + 3xy = 10$$

① $x = 3 - 2y$

sub ① into ②

$(3-2y)^2 + 3(3-2y)y = 10$

$9 - 12y + 4y^2 + 9y - 6y^2 = 10$

$0 = 2y^2 + 3y + 1$

$0 = (2y+1)(y+1)$

$y = -\frac{1}{2}$ $\quad y = -1$

$\Rightarrow x = 3 - 2(-\frac{1}{2})$

$x = 4$

$(4, -\frac{1}{2})$

$x = 3 - 2(-1)$

$x = 5$

$(5, -1)$

Test Your Understanding:

1. Solve the simultaneous equations: $3x^2 + y^2 = 21$ and $y = x + 1$

$3x^2 + (x+1)^2 = 21$

$3x^2 + x^2 + 2x + 1 = 21$

$4x^2 + 2x - 20 = 0$

$2x^2 + x - 10 = 0$

$(2x+5)(x-2) = 0$

$x = -\frac{5}{2}$ $\quad x = 2$

$y = -\frac{5}{2} + 1$

$= -\frac{5}{2} + \frac{2}{2}$

$= -\frac{3}{2}$

$(-\frac{5}{2}, -\frac{3}{2})$

$y = 2 + 1$

$y = 3$

$(2, 8)$

Exercise 3B Page 41

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Simultaneous Equations and Graphs

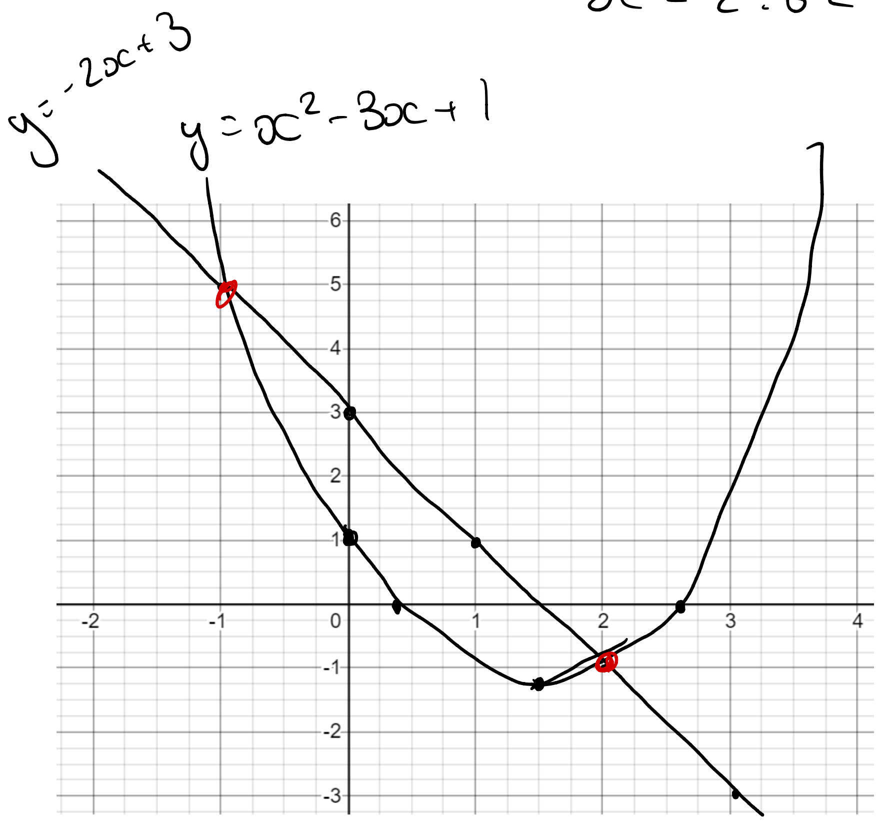
Examples:

1a. On the same axes, draw the graphs of $2x + y = 3$ and $y = x^2 - 3x + 1$

$y = -2x + 3$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$x = 2.62$ or $x = 0.382$



$y = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 1$

$y = \left(x - \frac{3}{2}\right)^2 - \frac{5}{4}$

$\left(\frac{3}{2}, -\frac{5}{4}\right)$

1b. Use your graph to write down the solutions to the simultaneous equations

$(-1, 5)$ $(2, -1)$

1c. What algebraic method could we have used to show the graphs would have intersected twice?

$y = -2x + 3$ $y = x^2 - 3x + 1$

$-2x + 3 = x^2 - 3x + 1$

$0 = x^2 - x - 2$

$b^2 - 4ac = (-1)^2 - 4(1)(-2)$

$= 1 + 8$

$= 9$

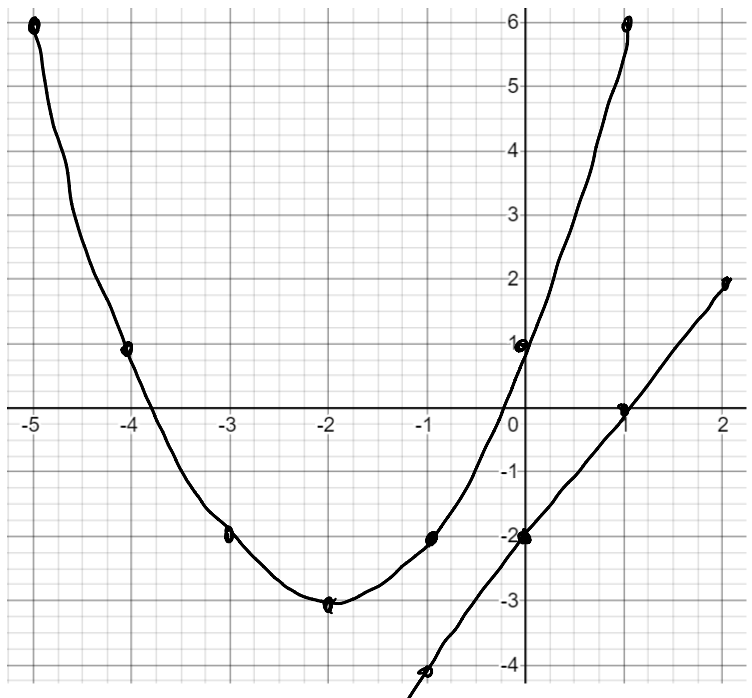
discriminant > 0 ie 2 real solutions

Example 2

a) On the same axes, draw the graphs of:

$$y = 2x - 2 \qquad y = x^2 + 4x + 1$$

x	-5	-4	-3	-2	-1	0	1	2
y	6	1	-2	-3	-2	1	6	13



no' intersections

$b^2 - 4ac < 0$

b) Prove algebraically that the lines never meet

$y = 2x - 2 \qquad y = x^2 + 4x + 1$

$2x - 2 = x^2 + 4x + 1$

$0 = x^2 + 2x + 3$

Discriminant $= (2)^2 - 4(1)(3)$

$= -8$

< 0 ie no real solution.

Test Your Understanding

The line with equation $y = 2x + 1$ meets the curve with equation $kx^2 + 2y + (k - 2) = 0$ at exactly one point. Given that k is a positive constant:

$2y = -kx^2 - (k-2)$

$y = -\frac{k}{2}x^2 - \frac{(k-2)}{2}$

a) Find the value of k. $b^2 - 4ac = 0$

b) For this value of k, find the coordinates of this point of intersection

a) $kx^2 + 2(2x+1) + (k-2) = 0$

$kx^2 + 4x + 2 + k - 2 = 0$

$kx^2 + 4x + k = 0$

$b^2 - 4ac = 4^2 - 4(k)(k)$

$0 = 16 - 4k^2$

$4k^2 = 16$

$k^2 = 4$

$k = \pm 2$

$\underline{\underline{k = 2}}$

b) $2x^2 + 2y = 0$

① $y = -x^2$

② $y = 2x + 1$

① = ②

$-x^2 = 2x + 1$

$0 = x^2 + 2x + 1$

$0 = (x+1)^2$

$x = -1$ repeated root.

② $y = 2(-1) + 1$

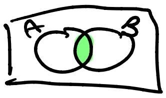
$y = -1$

$\underline{\underline{(-1, -1)}}$

Exercise 3C Page 45

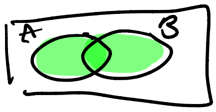
Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Set Builder Notation



Recap from GCSE:

- We use curly braces to list the values in a set, e.g. $A = \{1,4,6,7\}$
- If A and B are sets then $A \cap B$ is the **intersection** of A and B, giving a set which has the elements in A **and** B.



- $A \cup B$ is the **union** of A and B, giving a set which has the elements in A **or** in B.
- $\emptyset$ is the empty set, i.e. the set with nothing in it. → 1, 2, 3, 4 . . .
- Sets can also be infinitely large. $\mathbb{N}$ is the set of natural numbers (all positive integers), $\mathbb{Z}$ is the set → -3, 0, 7 of all integers (including negative numbers and 0) and $\mathbb{R}$ is the set of all real numbers (including all possible decimals). → 2.3, -1, 4, 0
- We write $x \in A$ to mean "x is a member of the set A". So $x \in \mathbb{R}$

Quick Fire Examples

$\{1,2,3\} \cap \{3,4,5\} = \{3\}$

$\{1,2,3\} \cup \{3,4,5\} = \{1, 2, 3, 4, 5\}$

$\{1,2\} \cap \{3,4\} \quad = \emptyset$

Examples: ← integers $\{$ie $-3, -2, -1, 0, 1, 2, 3\}$

1. $\{2x : x \in \mathbb{Z}\}$ → all even integers.

$\{\ldots -6, -4, -2, 0, 2, 4, 6 \ldots\}$

Natural numbers $\{1, 2, 3 \ldots\}$

2. $\{2^x : x \in \mathbb{N}\} = \{2, 4, 8, 16, 32, 64 \ldots\}$

$\{2, 3, 5, 7, 11, 13 \ldots\}$

3. $\{xy: \ x, y \ are \ prime\} = \{4, 6, 9, 10, 15, 14 \ldots\}$

Solving Inequalities

Linear inequalities Examples

1. $2x + 1 > 5$

$2x > 4$

$x > 2$

2. $3(x - 5) \geq 5 - 2(x - 8)$

$3x - 15 \geq 5 - 2x + 16$

$5x \geq 36$

$x \geq 7.2$

3. $-x \geq 2$

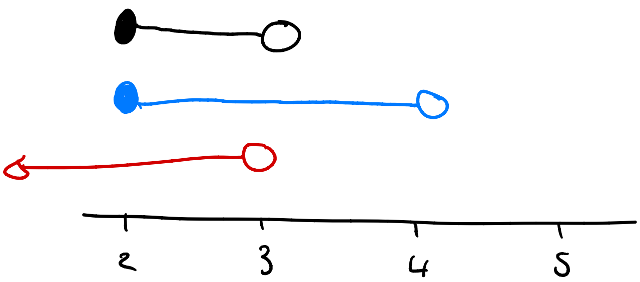
$x \leq -2$

Combining Inequalities

When combining inequalities always draw a number line to help!

Example:

If $x < 3$ and $2 \leq x < 4$, what is the combined solution set?



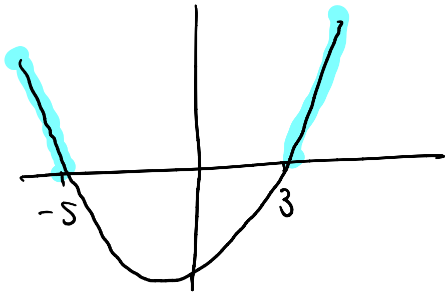
$2 \leq x < 3$

Quadratic Inequalities:

Examples

above x-axis

1. Solve $x^2 + 2x - 15 > 0$



$0 = (x+5)(x-3)$

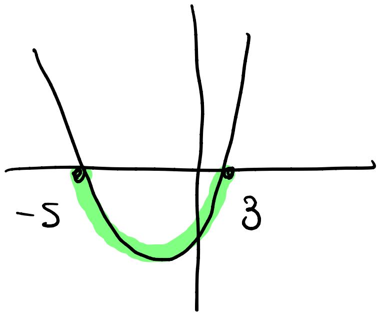
$x = -5 \qquad x = 3$

$\{x : x < -5 \cup x > 3\} \quad x \in \mathbb{R}$

below x-axis

2. Solve $x^2 + 2x - 15 \leq 0$

$\{x : -5 \leq x \leq 3\}, \quad x \in \mathbb{R}$

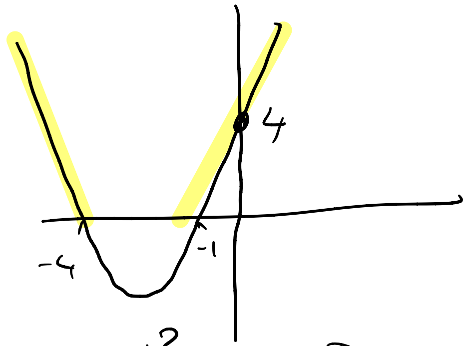


3. Solve $x^2 + 5x \geq -4$

$x^2 + 5x + 4 \geq 0$

$(x+4)(x+1) = 0$

$x = -4 \quad x = -1$



$\{x : x < -4 \cup x > -1\} \quad x \in \mathbb{R}$

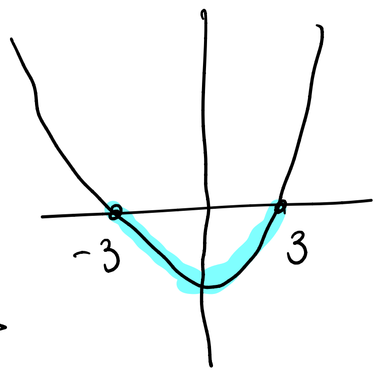
4. Solve $x^2 < 9$

$x^2 - 9 < 0$

$(x+3)(x-3) = 0$

$x = 3, \ x = -3$

$\{x : -3 < x < 3\} \quad x \in \mathbb{R}$



Test Your Understanding

Find the set of values of x for which

(*a*) $3(x - 2) < 8 - 2x$, **(2)**

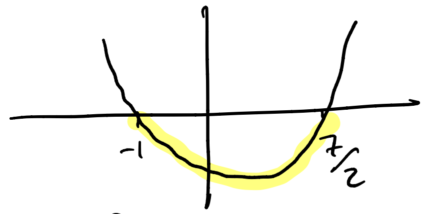
(*b*) $(2x - 7)(1 + x) < 0$, **(3)**

(*c*) both $3(x - 2) < 8 - 2x$ **and** $(2x - 7)(1 + x) < 0$. **(1)**

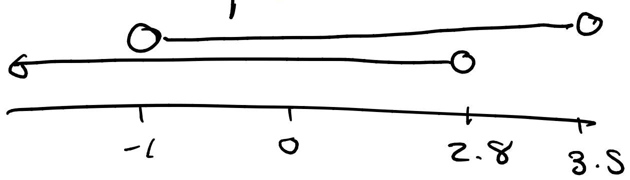
a) $3x - 6 < 8 - 2x$

$5x < 14$

$\{x : x < 2.8\}$ $x \in \mathbb{R}$



c)



b) $x = \frac{7}{2}$ $\quad x = -1$

$\{x : -1 < x < 3.5\}$ $x \in \mathbb{R}$

$\{x : -1 < x < 2.8\}$ $x \in \mathbb{R}$

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(*a*) show that $q^2 + 8q < 0$. **(2**)

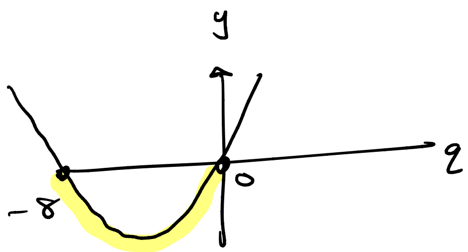
4

$b^2 - 4ac < 0$

(*b*) Hence find the set of possible values of q. **(3)**

a) $b^2 - 4ac \Rightarrow q^2 - 4(2q)(-1) < 0$

$\Rightarrow q^2 + 8q < 0$



b) $q(q + 8) = 0$

$q = 0 \quad q = -8$

$\{q : -8 < q < 0\}$ $q \in \mathbb{R}$

<u>Division by x</u>

We can't multiply x because it might be negative

Find the set of values for which $\frac{6}{x} > 2, \quad x \neq 0$

To get around this problem we multiply by x^2

$\frac{6}{x} > 2$

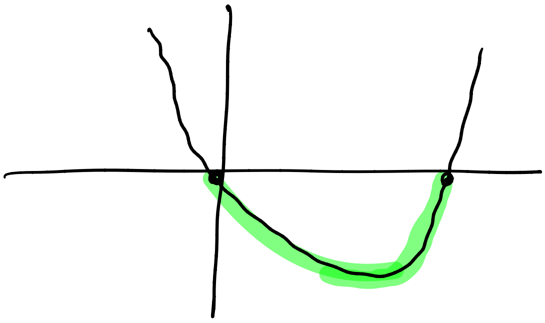
$6x > 2x^2$

$0 > 2x^2 - 6x$

$0 > x^2 - 3x$

$0 = x(x-3)$

$x = 0$ or $x = 3$



$x^2 - 3x < 0$

$\{x : 0 < x < 3\}\ x \in \mathbb{R}$

Exercise 3D/ 3E Page 47 – 51

Sketching Inequalities:

Examples

1. L_1 has equation $y = 12 + 4x$. L_2 has equation $y = x^2$.

The diagram shows a sketch of L_1 and L_2 on the same axes.

a) Find the coordinates of P_1 and P_2, the points of intersection.

b) Hence write down the solution to the inequality

$12 + 4x > x^2$.

① = ②

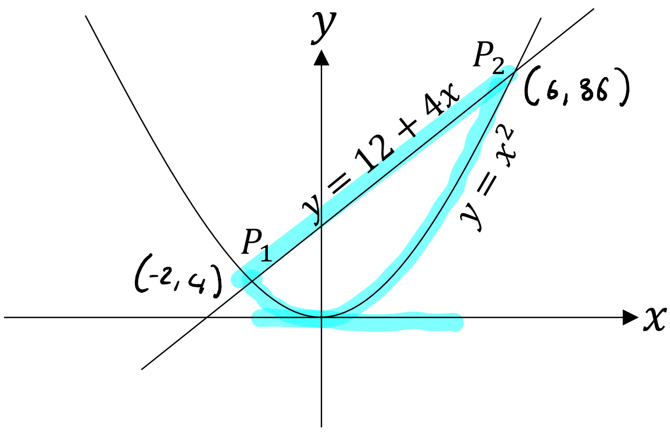
$12 + 4x = x^2$

$0 = x^2 - 4x - 12$

$0 = (x-6)(x+2)$

$x = 6$ $\quad x = -2$

$y = 36$ $\quad y = 4$



$\{x : -2 < x < 6\}$ $x \in \mathbb{R}$

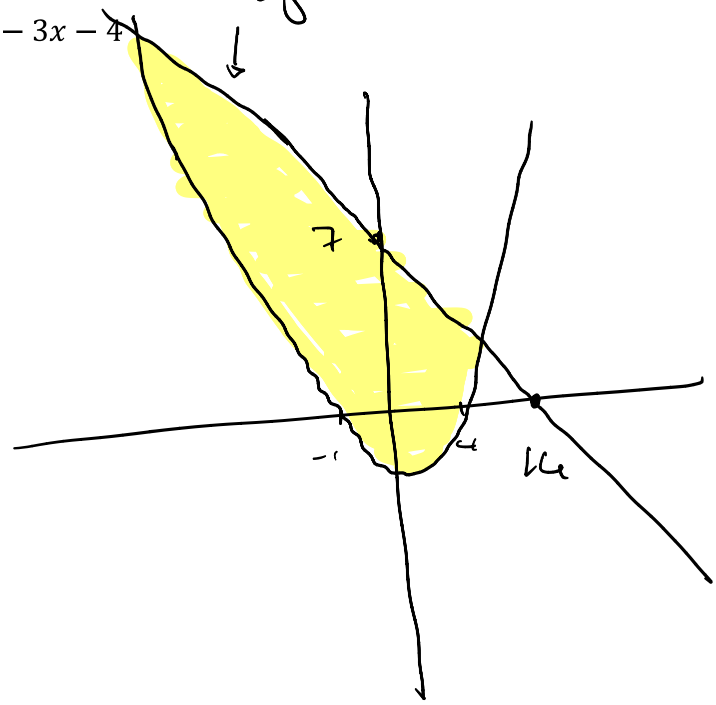
2. Shade the region that satisfies the inequalities:

$y < -\frac{1}{2}x + 7$

1 $2y + x < 14$

2 $y \geq x^2 - 3x - 4$

$2y + x = 14$



1 $y = -\frac{1}{2}x + 7$

$(0, 7)$

$y = 0 \quad 0 = -\frac{1}{2}x + 7$

$\frac{1}{2}x = 7$

$x = 4.$

$(14, 0)$

$y = x^2 - 3x - 4$

$y = (x - 4)(x + 1)$

$x = 4 \quad x = -1$

Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.

Exercise 3F/ 3G Page 53 – 55

Exam Style Questions

Q1.

Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$

(2)

(b) $3x^2 + 8x - 3 < 0$

(4)

(Total 6 marks)

Q2.

Find the set of values of x for which

(a) $4x - 3 > 7 - x$

(2)

(b) $2x^2 - 5x - 12 < 0$

(4)

(c) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$

(1)

(Total 7 marks)

Q3.

The equation

$$(k + 3)\,x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 24$$

(4)

(b) Hence find the set of possible values of k.

(3)

(Total 7 marks)

Q4.

Find the set of values of x for which

(a) $3x - 7 > 3 - x$

(2)

(b) $x^2 - 9x \leq 36$

(4)

(c) **both** $3x - 7 > 3 - x$ **and** $x^2 - 9x \leq 36$

(1)

(Total 7 marks)

Exam Style Questions

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Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$

(2)

(b) $3x^2 + 8x - 3 < 0$

(4)

(Total 6 marks)

Q2.
Find the set of values of x for which

(a) $4x - 3 > 7 - x$

(2)

(b) $2x^2 - 5x - 12 < 0$

(4)

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(1)
(Total 7 marks)

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(Total 7 marks)

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(1)

(Total 7 marks)

<u>Exam Style Questions</u>

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Find the set of values of x for which

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(Total 6 marks)

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(1)

(Total 7 marks)

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

<u>Exam Style Questions</u>

Q1.

Find the set of values of x for which

(a) $2(3x + 4) > 1 - x$

(2)

(b) $3x^2 + 8x - 3 < 0$

(4)

(Total 6 marks)

Q2.

Find the set of values of x for which

(a) $4x - 3 > 7 - x$

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The equation

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