Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Instruction – Transition Task

- Aim to complete this booklet independently
- If you need support, use the video/ written solutions provided on the Urmston Grammar website.
- Complete all exam questions at the end of each section and mark them using the mark scheme provided.
- You do not need to do anything with the exercise boxes ->

Exercise 1A Page 3

First few lessons at Urmston Grammar

Lesson 1 – You will hand in your transition work to your teacher and then revise chapters 1, 2 and 3 in preparation for your skills test.
Lesson 2 – You will complete a skills test on chapters 1, 2, and 3
Lesson 3 – You will start new content.

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Diagnostic for Chapter 1 Algebraic Expressions

1 Simplify:

a $4m^2n + 5mn^2 - 2m^2n + mn^2 - 3mn^2$

b $3x^2 - 5x + 2 + 3x^2 - 7x - 12$

← GCSE Mathematics

2 Write as a single power of 2:

a $2^5 \times 2^3$ **b** $2^6 \div 2^2$

c $(2^3)^2$ ← GCSE Mathematics

3 Expand:

a $3(x + 4)$ **b** $5(2 - 3x)$

c $6(2x - 5y)$ ← GCSE Mathematics

4 Write down the highest common factor of:

a 24 and 16 **b** $6x$ and $8x^2$

c $4xy^2$ and $3xy$ ← GCSE Mathematics

5 Simplify:

a $\frac{10x}{5}$ **b** $\frac{20x}{2}$ **c** $\frac{40x}{24}$

Basic Index Laws

Examples

Simplify $(a^3)^2 \times 2a^2$

2. Simplify $(4x^3y)^3$

3. Simplify $2x^2(3 + 5x) - x(4 - x^2)$

4. Simplify $\frac{x^3-2x}{3x^2}$ (2 methods)

Test Your Understanding:

1. Simplify $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$

Simplify $\frac{2x+x^5}{4x^3}$

3.Expand and simplify $2x(3-x^2)-4x^3(3-x)$

4.Simplify $2^x \times 3^x$

Exercise 1A Page 3

Negative and Fractional Indices

1. Prove that $x^{\frac{1}{2}} = \sqrt{x}$

2. Evaluate $27^{-\frac{1}{3}}$

3. Evaluate $32^{\frac{2}{5}}$

4. Simplify $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

Evaluate $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

6. If $b = \frac{1}{9}a^2$, determine $3b^{-2}$ in the form ka^n where k, n are constants

Exercise 1D Page 11

Brackets: Expanding

Example: $(x+1)(x+2)(x+3)$

Questions

Expand and simplify

$$(x+5)(x-2)(x+1)$$

Expand and simplify:

$$2(x-3)(x-4)$$

Expand and simplify:

$$(2x-1)^3$$

Exercise 1B Page 5

Brackets: Factorising

Examples:

1. $x^2 - 5x - 14$

2. $2x^2 + 5x - 12$

3. $4x^2 - 9$

4. $x^3 - x$

$x^3 + 3x^2 + 2x$

Test your understanding:

Factorise completely

1. $6x^2 + x - 2$

2. $x^3 - 7x^2 + 12x$

3. $x^4 - 1$

4. $x^3 - 1$

Exercise 1C Page 8

Surds:

Recap:

Simplify:

1. $\sqrt{3} \times 2$

2. $3\sqrt{5} \times 2\sqrt{5}$

3. $\sqrt{8}$

4. $\sqrt{12} + \sqrt{27}$

$(\sqrt{8} + 1)(\sqrt{2} - 3)$

Exercise 1E Page 11

Rationalising the denominator:

Examples:

1. $\frac{3}{\sqrt{2}}$

2. $\frac{6}{\sqrt{3}}$

$\frac{7}{\sqrt{7}}$

$\frac{15}{\sqrt{5}} + \sqrt{5}$

Test your understanding:

$\frac{12}{\sqrt{3}} =$

$\frac{2}{\sqrt{6}} =$

$\frac{4\sqrt{2}}{\sqrt{8}} =$

More Complicated Examples:

1. $\frac{1}{\sqrt{2}+1}$

2. $\frac{3}{\sqrt{6}-2}$

3. $\frac{4}{\sqrt{3}+1}$

4. $\frac{3\sqrt{2}+4}{5\sqrt{2}-7}$

Test Your Understanding:

Rationalise the denominator and simplify

$$\frac{4}{\sqrt{5}-2}$$

$$\frac{2\sqrt{3}-1}{3\sqrt{3}+1}$$

Solve $y(\sqrt{3}-1)=8$

Give your answer in the form $a+b\sqrt{3}$ where a and b are integers.

Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.

Exercise 1F Page 15

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a *a*

$\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}$

b *c* *b* *c*

$\frac{1}{4}$

x $x^{\frac{1}{4}}$

$\frac{5-2\sqrt{3}}{\sqrt{3}-1}$

p *q* *p* *q*

$(32)^{\frac{3}{5}}$

$\left(\frac{25x^4}{4}\right)^{\frac{1}{2}}$

$\frac{7+\sqrt{5}}{3+\sqrt{5}}$ *a* *b* *a* *b*

a *a*

a *b* *a* *b*

$\frac{30}{\sqrt{5}}$

c *c*

x *y* *y* *x*

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Diagnostic for Chapter 2 Quadratics

1 Solve the following equations:

a $3x + 6 = x - 4$

b $5(x + 3) = 6(2x - 1)$

c $4x^2 = 100$

d $(x - 8)^2 = 64$ ← GCSE Mathematics

2 Factorise the following expressions:

a $x^2 + 8x + 15$ **b** $x^2 + 3x - 10$

c $3x^2 - 14x - 5$ **d** $x^2 - 400$

3 Sketch the graphs of the following equations, labelling the points where each graph crosses the axes:

a $y = 3x - 6$ **b** $y = 10 - 2x$

c $x + 2y = 18$ **d** $y = x^2$

← GCSE Mathematics

4 Solve the following inequalities:

a $x + 8 < 11$ **b** $2x - 5 \geqslant 13$

c $4x - 7 \leqslant 2(x - 1)$ **d** $4 - x < 11$

← GCSE Mathematics

Solving Quadratic Equations

The 3 ways to solve a quadratic:

Recap:

By factorisation

1. $x^2 + 5x - 6 = 0$

Using the Quadratic Formula

2. $x^2 + 5x - 6 = 0$

Examples

1. $(x - 1)^2 = 5$

2. Solve $x - 6\sqrt{x} + 8 = 0$

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Test your understanding

1. $(x+3)^2 = x+5$

2. $(2x+1)^2 = 5$

3. $\sqrt{x+3} = x-3$

4. $2x + \sqrt{x} - 1 = 0$

Exercise 2A/ 2B Page 20/ 22

Solving by Completing the Square

Completing the Square form:

Worked Examples (a = 1):

1. $x^2 + 12x$

2. $x^2 + 8x$

3. $x^2 - 2x$

4. $x^2 - 6x + 7$

More complicated examples (a not equal to 1):

1. Express $2x^2 + 12x + 7$ in the form $a(x + b)^2 + c$

2. Express $5 - 3x^2 + 6x$ in the form $a - b(x + c)^2$

Test Your Understanding:

1. Express $3x^2 - 18x + 4$ in the form $a(x + b)^2 + c$

2. Express $20x - 5x^2 + 3$ in the form $a - b(x + c)^2$

Solving by Completing the Square:

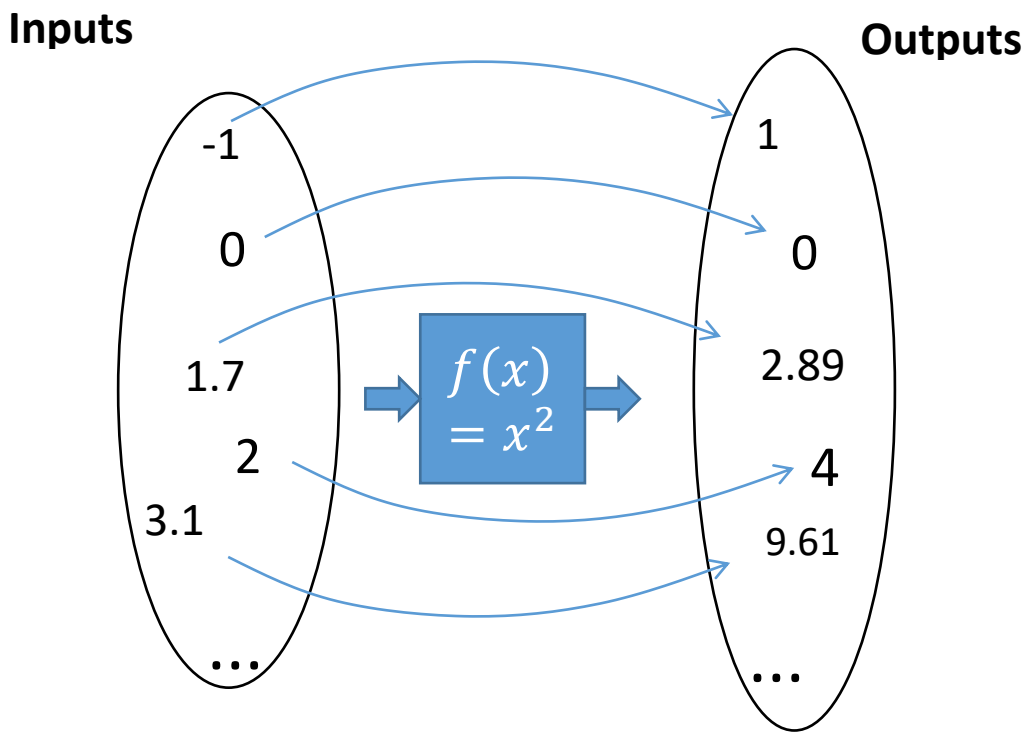
Note: Previously we factorised out the 3. This is because $3x^2 - 18x + 4$ on its own is an **expression,** so dividing by 3 (instead of factorising) would change the expression. However, in an equation, we can divide both sides by 3 without affecting the solutions.

Example

Solve the equation $3x^2 - 18x + 4 = 0$ by completing the square.

Exercise 2C/ 2D Page 23/ 24

Functions:



Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Examples:

1. If $f(x) = x^2 - 3x$ and $g(x) = x + 5, \quad x \in \mathbb{R}$

a) Find $f(-4)$

b) Find the values of x for which $f(x) = g(x)$

c) Find the roots of $f(x)$.

d) Find the roots of $g(x)$.

2. Determine the minimum value of the function $f(x) = x^2 - 6x + 2$, and state the value of x for which this minimum occurs.

Test Your Understanding:

$f(x)$	Completed square	Min/max value of $f(x)$	x for which this min/max occurs
$x^2 + 4x + 9$			
$x^2 - 10x + 21$			
$10 - x^2$			
$8 - x^2 + 6x$			

1. Find the minimum value of $f(x) = 2x^2 + 12x - 5$ and state the value of x for which this occurs.

2. Find the roots of the function $f(x) = 2x^2 + 3x + 1$

3. Find the roots of the function $f(x) = x^4 - x^2 - 6$

Exercise 2E Page 27

Quadratic Graphs:

Example: Sketch the graph of $y = x^2 + 3x - 4$ and find the coordinates of the turning point.

Example

Sketch the graph of $y = 4x - 2x^2 - 3$ and find the coordinates of the turning point. Write down the equation of the line of symmetry.

Test Your Understanding

Sketch the following, indicating any intercepts with the axis, the turning point and the equation of the line of symmetry.

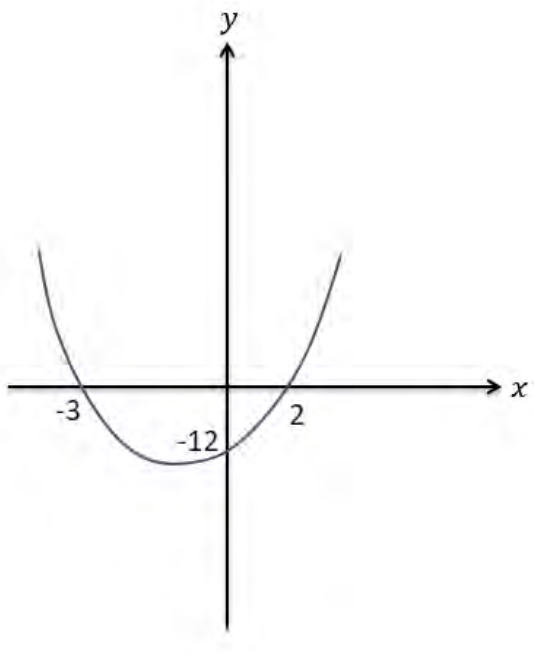
1. $y = x^2 + 4$

2. $y = x^2 - 7x + 10$

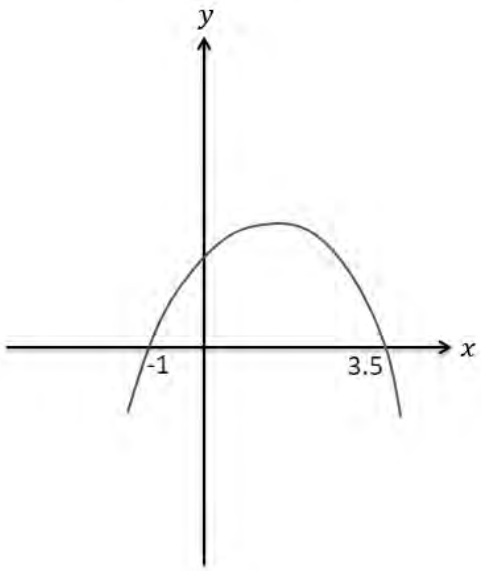
3. $y = 5x + 3 - 2x^2$

4. $y = x^2 + 4x + 11$

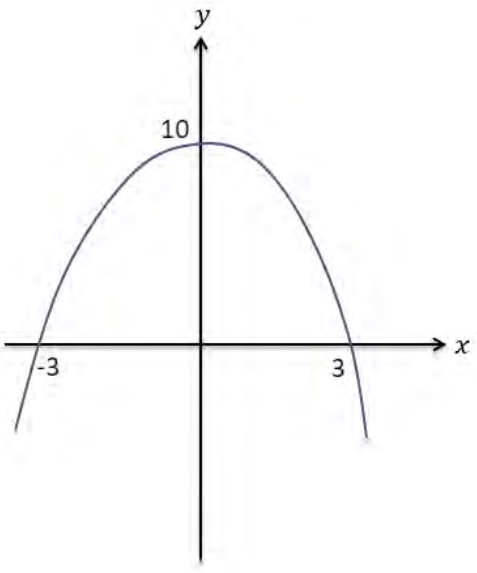
Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



Exercise 2F Page

The Discriminant

Quick fire questions:

Equation	Discriminant	No. of distinct real roots
$x^2 + 3x + 4 = 0$		
$x^2 - 4x + 1 = 0$		
$x^2 - 4x + 4 = 0$		
$2x^2 - 6x - 3 = 0$		
$x - 4 - 3x^2 = 0$		
$1 - x^2 = 0$		

Example:

8. The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(*a*) Find the value of p.

(4)

(*b*) For this value of p, solve the equation $x^2 + 2px + (3p + 4) = 0$.

(2)

Test Your Understanding:

1. $x^2 + 5kx + (10k + 5) = 0$ where k is a positive constant.

Given that this equation has equal roots, determine the value of k.

2. Find the range of values of k for which $x^2 + 6x + k = 0$ has two distinct real solutions.

Exercise 2G Page 32

Modelling

Example

A spear is thrown over level ground from the top of a tower.

The height, in metres, of the spear above the ground after t seconds is modelled by the function: $h(t) = 12.25 + 14.7t - 4.9t^2, \quad t \geq 0$

a) Interpret the meaning of the constant term 12.25 in the model.

b) After how many seconds does the spear hit the ground?

c) Write $h(t)$ in the form $A - B(t - C)^2$, where A, B and C are constants to be found.

d) Using your answer to part c or otherwise, find the maximum height of the spear above the ground, and the time at which this maximum height is reached?

Exercise 2H Page 34

Quadratics exam style question

A ball is thrown upwards from a rooftop 80m above the ground. It will reach a maximum vertical height and then fall back to the ground.

The height of the ball from ground at time t is h, given by the formula:

$h = -16t^2 + 64t + 80$

a) Calculate the height reached by the ball after 1 second.

b) Calculate the maximum height reached by the ball and after how many seconds from when it is thrown this maximum height is reached.

c) Calculate how long will it take before the ball hits the ground.

Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.

p *x* *x* *p* *p*

p *p* *p*

p

$$x\sqrt{2} - \sqrt{18} = x$$

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

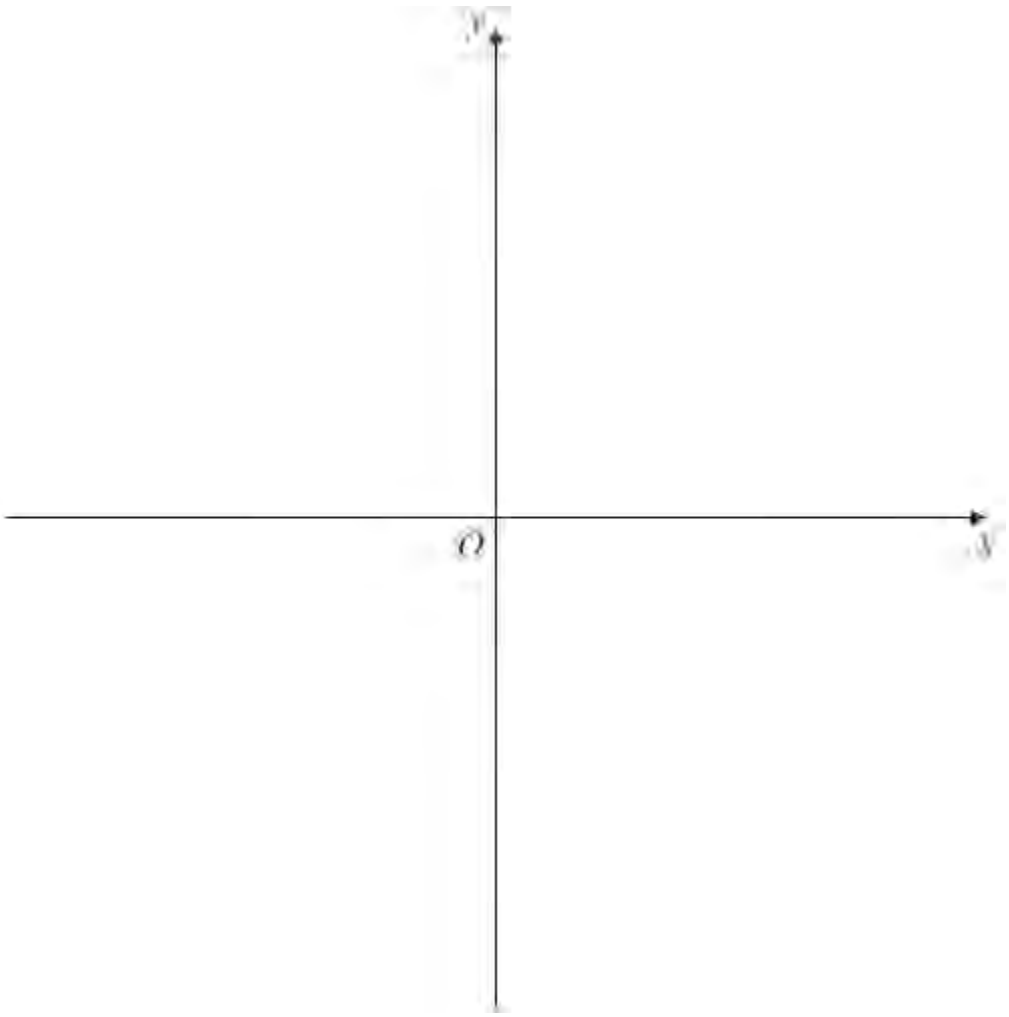
$$4x - 5 - x^2 = q - (x + p)^2$$

p *q*

p *q*

x *x*

y *x* *x*



$f(x) = x^2 + (k+3)x + k$

k

x *k*

x *k* *a* *b* *a* *b*

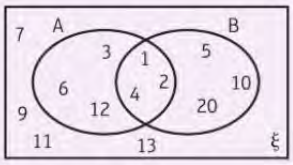
k *x*

x *px* *p* *p*

p

Diagnostic for Chapter 3 Equations and Inequalities

1 A = {factors of 12}
B = {factors of 20}
Write down the numbers in each of these sets:



a $A \cap B$ **b** $(A \cup B)'$

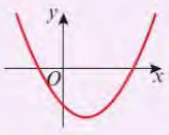
2 Simplify these expressions.

a $\sqrt{75}$ **b** $\dfrac{2\sqrt{45}+3\sqrt{32}}{6}$

3 Match the equations to the correct graph. Label the points of intersection with the axes and the coordinates of the turning point.

a $y = 9 - x^2$ **b** $y = (x-2)^2 + 4$
c $y = (x-7)(2x+5)$

i



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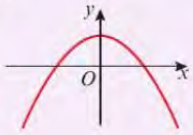

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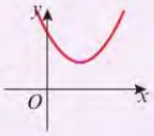


Simultaneous Equations

Simultaneous Equations Solution Sets

Scenario	Example	Solution Set
A single solution:	$x + y = 9$ $x - y = 1$	
Two solutions:	$x^2 + y^2 = 10$ $x + y = 4$	
No solutions:	$x + y = 1$ $x + y = 3$	
Infinitely large set of solutions:	$x + y = 1$ $2x + 2y = 2$	

Example (You can do this on your calculator!)

Solve the simultaneous equations

$$3x + y = 8$$

$$2x - 3y = 9$$

Method 1 : Elimination

Method 2: Substitution

Exercise 3A Page 40

Linear and Quadratic

Example:

Solve the simultaneous equations:

$$x + 2y = 3$$

$$x^2 + 3xy = 10$$

Test Your Understanding:

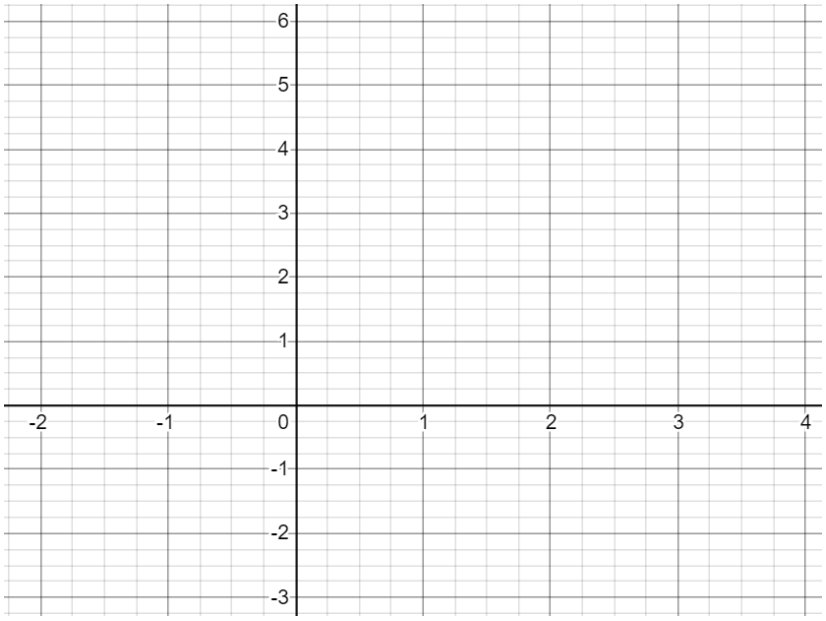
1. Solve the simultaneous equations: $3x^2 + y^2 = 21$ and $y = x + 1$

Exercise 3B Page 41

Simultaneous Equations and Graphs

Examples:

1a. On the same axes, draw the graphs of $2x + y = 3$ and $y = x^2 - 3x + 1$



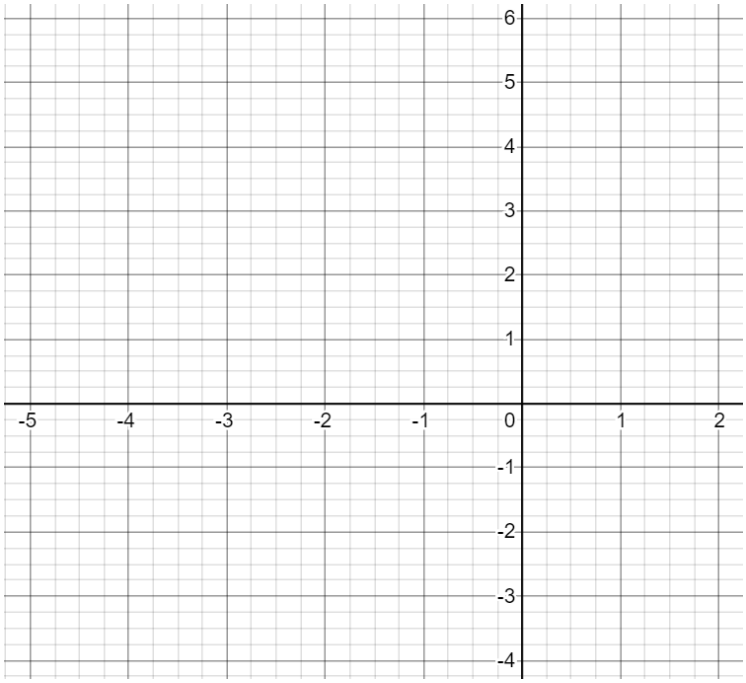
1b. Use your graph to write down the solutions to the simultaneous equations

1c. What algebraic method could we have used to show the graphs would have intersected twice?

Example 2

a) On the same axes, draw the graphs of:

$$y = 2x - 2 \qquad y = x^2 + 4x + 1$$



b) Prove algebraically that the lines never meet

Test Your Understanding

The line with equation $y = 2x + 1$ meets the curve with equation $kx^2 + 2y + (k - 2) = 0$ at exactly one point. Given that k is a positive constant:

a) Find the value of k.

b) For this value of k, find the coordinates of this point of intersection

Exercise 3C Page 45

Set Builder Notation

Recap from GCSE:

We use curly braces to list the values in a set, e.g. $A = \{1,4,6,7\}$
If A and B are sets then $A \cap B$ is the **intersection** of A and B, giving a set which has the elements in A **and** B.
$A \cup B$ is the **union** of A and B, giving a set which has the elements in A **or** in B.
$\emptyset$ is the empty set, i.e. the set with nothing in it.
Sets can also be infinitely large. $\mathbb{N}$ is the set of natural numbers (all positive integers), $\mathbb{Z}$ is the set of all integers (including negative numbers and 0) and $\mathbb{R}$ is the set of all real numbers (including all possible decimals).
We write $x \in A$ to mean "x is a member of the set A". So $x \in \mathbb{R}$

Quick Fire Examples

$\{1,2,3\} \cap \{3,4,5\} =$

$\{1,2,3\} \cup \{3,4,5\} =$

$\{1,2\} \cap \{3,4\} \quad =$

Examples:

1. $\{2x : x \in \mathbb{Z}\}$

2. $\{2^x : x \in \mathbb{N}\}$

3. $\{xy\colon\ x, y\ are\ prime\}$

Solving Inequalities

Linear inequalities Examples

1. $2x + 1 > 5$

2. $3(x - 5) \geq 5 - 2(x - 8)$

3. $-x \geq 2$

Combining Inequalities

When combining inequalities always draw a number line to help!

Example:

If $x < 3$ and $2 \leq x < 4$, what is the combined solution set?

Quadratic Inequalities:

Examples

1. Solve $x^2 + 2x - 15 > 0$

2. Solve $x^2 + 2x - 15 \leq 0$

3. Solve $x^2 + 5x \geq -4$

4. Solve $x^2 < 9$

Test Your Understanding

Find the set of values of x for which

(*a*) $3(x-2) < 8-2x$,

(2)

(*b*) $(2x-7)(1+x) < 0$,

(3)

(*c*) both $3(x-2) < 8-2x$ **and** $(2x-7)(1+x) < 0$.

(1)

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(*a*) show that $q^2 + 8q < 0$.

(2)

(*b*) Hence find the set of possible values of q.

(3)

Division by x

Find the set of values for which $\frac{6}{x} > 2, \quad x \neq 0$

Exercise 3D/ 3E Page 47 – 51

Sketching Inequalities:

Examples

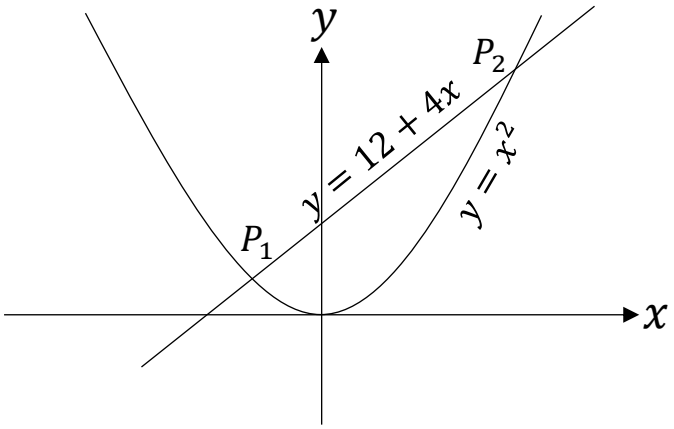
1. L_1 has equation $y = 12 + 4x$. L_2 has equation $y = x^2$.

The diagram shows a sketch of L_1 and L_2 on the same axes.

a) Find the coordinates of P_1 and P_2, the points of intersection.

b) Hence write down the solution to the inequality

$12 + 4x > x^2$.



2. Shade the region that satisfies the inequalities:

$$2y + x < 14$$

$$y \geq x^2 - 3x - 4$$

Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.

Exercise 3F/ 3G Page 53 – 55

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x

x *x*

x *x*

x

x *x*

x *x*

x *x* *x* *x*

k *x* *x* *k* *k*

x

k

k *k*

k

x

x *x*

x *x*

x *x* *x* *x*