Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Instruction – Transition Task

- Aim to complete this booklet independently
- If you need support, use the video/ written solutions provided on the Urmston Grammar website.
- Complete all exam questions at the end of each section and mark them using the mark scheme provided.
- You do not need to do anything with the exercise boxes ->

Exercise 1A Page 3

First few lessons at Urmston Grammar

Lesson 1 – You will hand in your transition work to your teacher and then revise chapters 1, 2 and 3 in preparation for your skills test.
Lesson 2 – You will complete a skills test on chapters 1, 2, and 3
Lesson 3 – You will start new content.

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Diagnostic for Chapter 1 Algebraic Expressions

1 Simplify:

a $4m^2n + 5mn^2 - 2m^2n + mn^2 - 3mn^2$

b $3x^2 - 5x + 2 + 3x^2 - 7x - 12$

← **GCSE Mathematics**

2 Write as a single power of 2:

a $2^5 \times 2^3$ **b** $2^6 \div 2^2$

c $(2^3)^2$ ← **GCSE Mathematics**

3 Expand:

a $3(x + 4)$ **b** $5(2 - 3x)$

c $6(2x - 5y)$ ← **GCSE Mathematics**

4 Write down the highest common factor of:

a 24 and 16 **b** $6x$ and $8x^2$

c $4xy^2$ and $3xy$ ← **GCSE Mathematics**

5 Simplify:

a $\frac{10x}{5}$ **b** $\frac{20x}{2}$ **c** $\frac{40x}{24}$

Transition Task. Chapter 1 - Algebraic Expressions.
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Basic Index Laws

Examples

Simplify $(a^3)^2 \times 2a^2$

2. Simplify $(4x^3y)^3$

3. Simplify $2x^2(3 + 5x) - x(4 - x^2)$

4. Simplify $\frac{x^3-2x}{3x^2}$ (2 methods)

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Test Your Understanding:

1. Simplify $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$

Simplify $\frac{2x+x^5}{4x^3}$

3.Expand and simplify $2x(3-x^2)-4x^3(3-x)$

4.Simplify $2^x \times 3^x$

Exercise 1A Page 3

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Negative and Fractional Indices

1. Prove that $x^{\frac{1}{2}} = \sqrt{x}$

2. Evaluate $27^{-\frac{1}{3}}$

3. Evaluate $32^{\frac{2}{5}}$

4. Simplify $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

Evaluate $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

6. If $b = \frac{1}{9}a^2$, determine $3b^{-2}$ in the form ka^n where k, n are constants

Exercise 1D Page 11

Brackets: Expanding

Example: $(x+1)(x+2)(x+3)$

Questions

Expand and simplify

$$(x+5)(x-2)(x+1)$$

Expand and simplify:

$$2(x-3)(x-4)$$

Expand and simplify:

$$(2x-1)^3$$

Exercise 1B Page 5

Brackets: Factorising

Examples:

1. $x^2 - 5x - 14$

2. $2x^2 + 5x - 12$

3. $4x^2 - 9$

4. $x^3 - x$

$x^3 + 3x^2 + 2x$

Test your understanding:

Factorise completely

1. $6x^2 + x - 2$

2. $x^3 - 7x^2 + 12x$

3. $x^4 - 1$

4. $x^3 - 1$

Exercise 1C Page 8

Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Surds:

Recap:

Simplify:

1. $\sqrt{3} \times 2$

2. $3\sqrt{5} \times 2\sqrt{5}$

3. $\sqrt{8}$

4. $\sqrt{12} + \sqrt{27}$

$$\left(\sqrt{8} + 1\right)\left(\sqrt{2} - 3\right)$$

Exercise 1E Page 11

Rationalising the denominator:

Examples:

1. $\frac{3}{\sqrt{2}}$

2. $\frac{6}{\sqrt{3}}$

$\frac{7}{\sqrt{7}}$

$\frac{15}{\sqrt{5}} + \sqrt{5}$

Test your understanding:

$\frac{12}{\sqrt{3}} =$

$\frac{2}{\sqrt{6}} =$

$\frac{4\sqrt{2}}{\sqrt{8}} =$

More Complicated Examples:

1. $\frac{1}{\sqrt{2}+1}$

2. $\frac{3}{\sqrt{6}-2}$

3. $\frac{4}{\sqrt{3}+1}$

4. $\frac{3\sqrt{2}+4}{5\sqrt{2}-7}$

Test Your Understanding:

Rationalise the denominator and simplify

$$\frac{4}{\sqrt{5}-2}$$

$$\frac{2\sqrt{3}-1}{3\sqrt{3}+1}$$

Solve $y\left(\sqrt{3}-1\right)=8$
Give your answer in the form $a+b\sqrt{3}$ where a and b are integers.

Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.

Exercise 1F Page 15

a *a*

$\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}$

b *c* *b* *c*

$\frac{1}{4}$

x $x^{\frac{1}{4}}$

$\frac{5-2\sqrt{3}}{\sqrt{3}-1}$

p *q* *p* *q*

$(32)^{\frac{3}{5}}$

$\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$

$\frac{7+\sqrt{5}}{3+\sqrt{5}}$ *a* *b* *a* *b*

a *a*

a *b* *a* *b*

$\frac{30}{\sqrt{5}}$

c *c*

x *y* *y* *x*

Question	Scheme	Marks
(a)	$\sqrt{32} = 4\sqrt{2}$ or $\sqrt{18} = 3\sqrt{2}$	B1
	$(\sqrt{32}+\sqrt{18} =)$ $\underline{7\sqrt{2}}$	B1 (2)
(b)	$\times\frac{3-\sqrt{2}}{3-\sqrt{2}}$ or $\times\frac{-3+\sqrt{2}}{-3+\sqrt{2}}$ seen	M1
	$\left[\frac{\sqrt{32}+\sqrt{18}}{3+\sqrt{2}}\times\frac{3-\sqrt{2}}{3-\sqrt{2}}=\right]$ $\frac{a\sqrt{2}(3-\sqrt{2})}{[9-2]}$ $\rightarrow$ $\frac{3a\sqrt{2}-2a}{[9-2]}$ (or better)	dM1
	$= \underline{3\sqrt{2}, -2}$	A1, A1 (4)
ALT	$(b\sqrt{2}+c)(3+\sqrt{2}) = 7\sqrt{2}$ leading to: $3b + c = 7$, $3c + 2b = 0$	M1
	e.g. $3(7-3b)+2b = 0$ (o.e.)	dM1
		6 marks
	Notes	
(a)	1st B1 for either surd simplified 2nd B1 for $7\sqrt{2}$ or accept $a = 7$. Answer only scores B1B1 NB Common error is $\sqrt{32}+\sqrt{18} = \sqrt{50} = 5\sqrt{2}$ this scores B0B0 but can use their "5" in (b) to get M1M1	
(b)	1st M1 for an attempt to multiply by $\frac{3-\sqrt{2}}{3-\sqrt{2}}$ (o.e.) Allow poor use of brackets 2nd dM1 for using $a\sqrt{2}$ to correctly obtain a numerator of the form $p + q\sqrt{2}$ where p and q are non-zero integers. Allow arithmetic slips e.g. $21\sqrt{2} - 28$ or $3\sqrt{2}\times\sqrt{2} = 3$ Follow through their $a = 7$ or a new value found in (b). Ignore denominator. Allow use of letter a. Dependent on 1st M1 So $3\sqrt{32} - \sqrt{64} + 3\sqrt{8} - \sqrt{36}$ is M0 until they reduce $p + q\sqrt{2}$ 1st A1 for $3\sqrt{2}$ or accept $b = 3$ from correct working 2nd A1 for -2 or accept $c = -2$ from correct working	
ALT	**Simultaneous Equations** 1st M1 for $(b\sqrt{2}+c)(3+\sqrt{2}) = 7\sqrt{2}$ and forming 2 simultaneous equations. Ft their $a = 7$ 2nd dM1 for solving their simultaneous equations: reducing to a linear equation in one variable	

Question Number	Scheme	Marks
(a)	$16^{\frac{1}{4}} = 2$ or $\frac{1}{16^{\frac{1}{4}}}$ or better	M1
	$\left(16^{-\frac{1}{4}} =\right)$ $\frac{1}{2}$ or 0.5 (ignore $\pm$)	A1
		(2)
(b)	$\left(2x^{-\frac{1}{4}}\right)^4 = 2^4 x^{-\frac{4}{4}}$ or $\frac{2^4}{x^{\frac{4}{4}}}$ or equivalent	M1
	$x\left(2x^{-\frac{1}{4}}\right)^4 = 2^4$ or 16	A1 cao
		(2) 4
	Notes	
(a)	M1 for a correct statement dealing with the $\frac{1}{4}$ or the $-$ power This may be awarded if 2 is seen or for reciprocal of their $16^{\frac{1}{4}}$ s.c ¼ is M1 A0 , also 2^{-1} is M1 A0 $\pm\frac{1}{2}$ is not penalised so M1 A1	
(b)	M1 for correct use of the power 4 on both the 2 and the x terms A1 for cancelling the x and simplifying to one of these two forms. Correct answers with no working get full marks	

Question Number	Scheme	Marks
	$\frac{5-2\sqrt{3}}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$	M1
	$= \frac{---}{2}$ denominator of 2	A1
	Numerator $= 5\sqrt{3}+5-2\sqrt{3}\sqrt{3}-2\sqrt{3}$	M1
	So $\frac{5-2\sqrt{3}}{\sqrt{3}-1} = -\frac{1}{2}+\frac{3}{2}\sqrt{3}$	A1
		4
	Alternative: $\left(p+q\sqrt{3}\right)(\sqrt{3}-1) = 5-2\sqrt{3}$, and form simultaneous equations in p and q	M1
	$-p+3q=5$ and $p-q=-2$	A1
	Solve simultaneous equations to give $p=-\frac{1}{2}$ and $q=\frac{3}{2}$.	M1 A1
	Notes	
	1st M1 for multiplying numerator and denominator by same correct expression 1st A1 for a correct denominator as a single number (NB depends on M mark) 2nd M1 for an attempt to multiply the numerator by $\left(\sqrt{3}\pm1\right)$ and get 4 terms with at least 2 correct. 2nd A1 for the answer as written or $p=-\frac{1}{2}$ and $q=\frac{3}{2}$. Allow -0.5 and 1.5. (Apply isw if correct answer seen, then slip writing $p=$, $q=$)	
	Answer only (very unlikely) is full marks if correct – no part marks	

Question Number	Scheme		Marks
(a)	$\left\{(32)^{\frac{3}{5}}\right\} = \left(\sqrt[5]{32}\right)^3$ or $\sqrt[5]{(32)^3}$ or 2^3 or $\sqrt[5]{32768}$		M1
	$= 8$		A1
			[2]
(b)	$\left\{\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}\right\} = \left(\frac{4}{25x^4}\right)^{\frac{1}{2}}$ or $\left(\frac{5x^2}{2}\right)^{-1}$ or $\frac{1}{\left(\frac{25x^4}{4}\right)^{\frac{1}{2}}}$	*See notes below*	M1
	$= \frac{2}{5x^2}$ or $\frac{2}{5}x^{-2}$	*See notes for other alternatives*	A1
			[2] 4

Notes

(a)

M1: for a full correct interpretation of the fractional power. **Note:** $5\times(32)^3$ is M0.

A1: for 8 only.

Note: Award M1A1 for writing down 8.

(b)

M1: For use of $\frac{1}{2}$ OR use of -1

Use of $\frac{1}{2}$: Candidate needs to demonstrate the they have rooted all three elements in their bracket.

Use of -1: Either Candidate has $\frac{1}{\text{Bracket}}$ or $\left(\frac{Ax^C}{B}\right)$ becomes $\left(\frac{B}{Ax^C}\right)$.

Allow M1 for...

- $\left(\frac{4}{25x^4}\right)^{\frac{1}{2}}$ or $\left(\frac{5x^2}{2}\right)^{-1}$ or $\frac{1}{\left(\frac{25x^4}{4}\right)^{\frac{1}{2}}}$ or $\sqrt{\left(\frac{4}{25x^4}\right)}$ or $\frac{1}{\sqrt{\left(\frac{25x^4}{4}\right)}}$ or $\left(\frac{\frac{1}{25x^4}}{\frac{1}{4}}\right)^{\frac{1}{2}}$ or $\frac{\frac{1}{5x^2}}{\frac{1}{2}}$ or $\frac{\frac{1}{5}x^{-2}}{\frac{1}{2}}$

  or $-\left(\frac{5x^2}{2}\right)$ or $\left(\frac{-5x^{-2}}{-2}\right)$ or $-\left(\frac{5x^{-2}}{2}\right)$ or $\frac{5x^{-2}}{2}$
- $\left(\frac{4}{25x^4}\right)^K$ or $\left(\frac{5x^2}{2}\right)^C$ where K, C are any powers including 1.

A1: for either $\frac{2}{5x^2}$ or $\frac{2}{5}x^{-2}$ or $0.4x^{-2}$ or $\frac{0.4}{x^2}$.

Note: $\left(\sqrt{\left(\frac{25x^4}{4}\right)}\right)^{-1}$ is not enough work by itself for the method mark.

Note: A final answer of $\frac{1}{\frac{5}{2}x^2}$ or $\frac{1}{2\frac{1}{2}x^2}$ or $\frac{1}{2.5x^2}$ is A0.

Note: Also allow $\pm\frac{2}{5x^2}$ or $\pm\frac{2}{5}x^{-2}$ or $\pm 0.4x^{-2}$ or $\pm\frac{0.4}{x^2}$ for A1.

Question number	Scheme	Marks
	(a) $(7+\sqrt{5})(3-\sqrt{5}) = 21-5+3\sqrt{5}-7\sqrt{5}$ Expand to get 3 or 4 terms $=16,\ -4\sqrt{5}$ (1st A for 16, 2nd A for $-4\sqrt{5}$) (i.s.w. if necessary, e.g. $16-4\sqrt{5} \rightarrow 4-\sqrt{5}$)	M1 A1, A1 (3)
	(b) $\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ (This is sufficient for the M mark) Correct denominator without surds, i.e. $9-5$ or 4 $4-\sqrt{5}$ or $4-1\sqrt{5}$	M1 A1 A1 (3) **[6]**
	(a) M1: Allowed for an attempt giving 3 or 4 terms, with at least 2 correct (even if unsimplified). e.g. $21-\sqrt{5}^2+\sqrt{15}$ scores M1. Answer only: $16-4\sqrt{5}$ scores full marks One term correct scores the M mark by implication, e.g. $26-4\sqrt{5}$ scores M1 A0 A1 (b) Answer only: $4-\sqrt{5}$ scores full marks One term correct scores the M mark by implication, e.g. $4+\sqrt{5}$ scores M1 A0 A0 $16-\sqrt{5}$ scores M1 A0 A0 Ignore subsequent working, e.g. $4-\sqrt{5}$ so $a=4,\ b=1$ Note that, as always, A marks are dependent upon the preceding M mark, so that, for example, $\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3+\sqrt{5}}{3-\sqrt{5}} = \frac{\ldots\ldots}{4}$ is M0 A0. Alternative $(a+b\sqrt{5})(3+\sqrt{5}) = 7+\sqrt{5}$, then form simultaneous equations in a and b. M1 Correct equations: $3a+5b=7$ and $3b+a=1$ A1 $a=4$ and $b=-1$ A1	

Question Number	Scheme	Marks
Q (a)	$(3\sqrt{7})^2 = 63$	B1 (1)
(b)	$(8+\sqrt{5})(2-\sqrt{5}) = 16-5+2\sqrt{5}-8\sqrt{5}$	M1
	$= 11, \ -6\sqrt{5}$	A1, A1
		(3)
		[4]
(a)	B1 for 63 only	
(b)	M1 for an attempt to expand <u>their</u> brackets with ≥ 3 terms correct. They may collect the $\sqrt{5}$ terms to get $16-5-6\sqrt{5}$ Allow $-\sqrt{5}\times\sqrt{5}$ or $-(\sqrt{5})^2$ or $-\sqrt{25}$ instead of the -5 These 4 values may appear in a list or table but they should have minus signs included **The next two marks should be awarded for the final answer but check that correct values follow from correct working. Do not use ISW rule** 1st A1 for 11 from $16-5$ <u>or</u> $-6\sqrt{5}$ from $-8\sqrt{5}+2\sqrt{5}$ 2nd A1 for <u>both</u> 11 and $-6\sqrt{5}$ <u>S.C - Double sign error in expansion</u> For $16-5-2\sqrt{5}+8\sqrt{5}$ leading to 11 + … allow <u>one</u> mark	

Question Number	Scheme	Marks
Q	$32 = 2^5$ or $2048 = 2^{11}$, $\sqrt{2} = 2^{1/2}$ or $\sqrt{2048} = (2048)^{\frac{1}{2}}$	B1, B1
	$a = \frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$	B1
		[3]
	1st B1 for $32 = 2^5$ or $2048 = 2^{11}$ This should be explicitly seen: $32\sqrt{2} = 2^a$ followed by $2^5\sqrt{2} = 2^a$ is OK Even writing $32\times 2 = 2^5\times 2(=2^6)$ is OK but simply writing $32\times 2 = 2^6$ is NOT 2nd B1 for $2^{\frac{1}{2}}$ or $(2048)^{\frac{1}{2}}$ seen. This mark may be implied 3rd B1 for answer as written. **Need a = …** so $2^{\frac{11}{2}}$ is B0 $a = \frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$ with no working scores full marks. If a = 5.5 seen then award 3/3 unless it is clear that the value follows from totally incorrect work. Part solutions: e.g. $2^5\sqrt{2}$ scores the first B1. <u>Special case</u>: If $\sqrt{2} = 2^{1/2}$ is not explicitly seen, but the final answer includes $\frac{1}{2}$, e.g. $a = 2\frac{1}{2}$, $a = 4\frac{1}{2}$, the second B1 is given by implication.	

Question Number	Scheme	Marks
(i)	$(5-\sqrt{8})(1+\sqrt{2})$	
	$=5+5\sqrt{2}-\sqrt{8}-4$	M1
	$=5+5\sqrt{2}-2\sqrt{2}-4$ $\sqrt{8}=2\sqrt{2}$, seen or implied at any point	B1
	$=1+3\sqrt{2}$ $1+3\sqrt{2}$ or $a=1$ and $b=3$.	A1 [3]
(ii)	**Method 1** — **Either** $\sqrt{80}+\frac{30}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$; **Method 2** — **Or** $\left(\frac{\sqrt{400}+30}{\sqrt{5}}\right)\frac{\sqrt{5}}{\sqrt{5}}$; **Method 3** — $\sqrt{80}+\frac{\sqrt{900}}{\sqrt{5}}=\sqrt{80}+\sqrt{180}$	M1
	Method 1: $=4\sqrt{5}+\ldots$; Method 2: $=\left(\frac{20+\ldots}{\ldots}\right)\frac{\ldots}{\ldots}$; Method 3: $=4\sqrt{5}+\ldots$	B1
	Method 1: $=4\sqrt{5}+6\sqrt{5}$; Method 2: $=\left(\frac{50\sqrt{5}}{5}\right)$; Method 3: $=4\sqrt{5}+6\sqrt{5}$	
	$=10\sqrt{5}$	A1 [3]
Alternative for (i)	$(5-2\sqrt{2})(1+\sqrt{2})$ This earns the B1 mark	
	$=5+5\sqrt{2}-2\sqrt{2}-2\sqrt{2}\sqrt{2}$ Multiplies out correctly with $2\sqrt{2}$. This may be seen or implied and may be simplified e.g. $=5+3\sqrt{2}-2\sqrt{4}$ o.e	M1
	For earlier use of $2\sqrt{2}$	B1
	$=1+3\sqrt{2}$ $1+3\sqrt{2}$ or $a=1$ and $b=3$.	A1 [3]
		6 marks

	Notes
(i)	**M1:** Multiplies out brackets correctly giving four correct terms or simplifying to correct expansion. (This may be implied by correct answer) – can appear as table **B1:** $\sqrt{8}=2\sqrt{2}$, seen or implied at any point **A1:** Fully and correctly simplified to $1+3\sqrt{2}$ or $a=1$ and $b=3$.
(ii)	**M1:** Rationalises denominator i.e. Multiplies $\left(\frac{k}{\sqrt{5}}\right)$ by $\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$ or $\left(\frac{-\sqrt{5}}{-\sqrt{5}}\right)$, seen or implied or uses Method 3 or similar e.g. $\left(\frac{30}{\sqrt{5}}\right)=\frac{6\times5}{\sqrt{5}}=6\sqrt{5}$ **B1:** (Independent mark) States $\sqrt{80}=4\sqrt{5}$ Or either $\sqrt{400}=20$ *or* $\sqrt{80}\sqrt{5}=20$ at any point if they use Method 2. **A1:** $10\sqrt{5}$ or $c=10$
	N.B There are other methods e.g. $\sqrt{80}=\frac{20}{\sqrt{5}}$ (B1) then add $\frac{20}{\sqrt{5}}+\frac{30}{\sqrt{5}}=\frac{50}{\sqrt{5}}$ then M1 A1as before Those who multiply initial expression by $\sqrt{5}$ to obtain $\sqrt{400}+30=20+30=50$ earn M0 B1 A0

Question Number	Scheme	Marks
	$(8^{2x+3} = (2^3)^{2x+3}) = 2^{3(2x+3)}$ or 2^{ax+b} with $a = 6$ or $b = 9$	M1
	$= 2^{6x+9}$ *or* $= 2^{3(2x+3)}$ as **final** answer with no errors or $(y =)6x + 9$ or $3(2x + 3)$	A1 [2]
		2 marks
	Notes	
	M1: Uses $8 = 2^3$, and **multiplies** powers $3(2x + 3)$. Does **not** add powers. (Just $8 = 2^3$ or $8^{\frac{1}{3}} = 2$ is M0) **A1:** Either 2^{6x+9} *or* $= 2^{3(2x+3)}$ or $(y =)6x + 9$ or $3(2x + 3)$	
	Note: Examples: 2^{6x+3} scores M1A0 : $8^{2x+3} = (2^3)^{2x+3} = 2^{3+2x+3}$ gets M0A0 **Special case:** : $= 2^{6x}2^9$ without seeing as single power M1A0 **Alternative method using logs:** $8^{2x+3} = 2^y \Rightarrow (2x+3)\log 8 = y \log 2 \Rightarrow y = \frac{(2x+3)\log 8}{\log 2}$ So $(y =)6x + 9$ or $3(2x + 3)$	M1 A1 [2]

Diagnostic for Chapter 2 Quadratics

1 Solve the following equations:

a $3x + 6 = x - 4$

b $5(x + 3) = 6(2x - 1)$

c $4x^2 = 100$

d $(x - 8)^2 = 64$

← **GCSE Mathematics**

2 Factorise the following expressions:

a $x^2 + 8x + 15$

b $x^2 + 3x - 10$

c $3x^2 - 14x - 5$

d $x^2 - 400$

3 Sketch the graphs of the following equations, labelling the points where each graph crosses the axes:

a $y = 3x - 6$

b $y = 10 - 2x$

c $x + 2y = 18$

d $y = x^2$

← **GCSE Mathematics**

4 Solve the following inequalities:

a $x + 8 < 11$

b $2x - 5 \geqslant 13$

c $4x - 7 \leqslant 2(x - 1)$

d $4 - x < 11$

← **GCSE Mathematics**

Solving Quadratic Equations

The 3 ways to solve a quadratic:

Recap:

By factorisation

1. $x^2 + 5x - 6 = 0$

Using the Quadratic Formula

2. $x^2 + 5x - 6 = 0$

Examples

1. $(x - 1)^2 = 5$

2. Solve $x - 6\sqrt{x} + 8 = 0$

Test your understanding

1. $(x+3)^2 = x+5$

2. $(2x+1)^2 = 5$

3. $\sqrt{x+3} = x-3$

4. $2x + \sqrt{x} - 1 = 0$

Exercise 2A/ 2B Page 20/ 22

Solving by Completing the Square

Completing the Square form:

Worked Examples (a = 1):

1. $x^2 + 12x$

2. $x^2 + 8x$

3. $x^2 - 2x$

4. $x^2 - 6x + 7$

More complicated examples (a not equal to 1):

1. Express $2x^2 + 12x + 7$ in the form $a(x+b)^2 + c$

2. Express $5 - 3x^2 + 6x$ in the form $a - b(x+c)^2$

Test Your Understanding:

1. Express $3x^2 - 18x + 4$ in the form $a(x+b)^2 + c$

2. Express $20x - 5x^2 + 3$ in the form $a - b(x+c)^2$

Solving by Completing the Square:

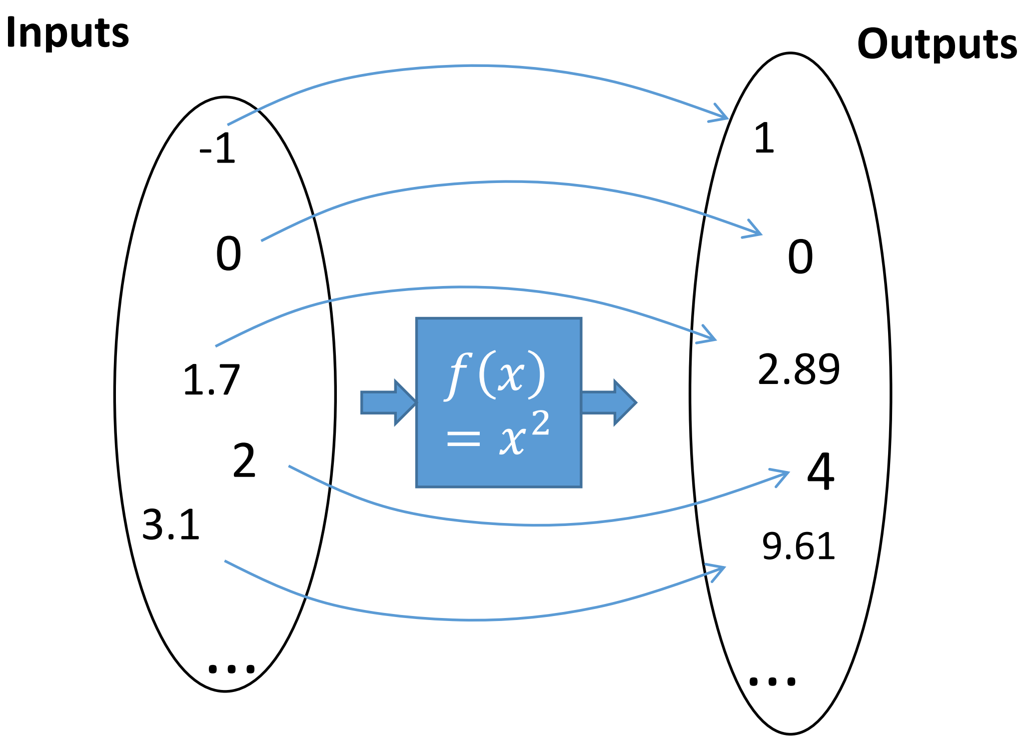
Note: Previously we factorised out the 3. This is because $3x^2 - 18x + 4$ on its own is an **expression,** so dividing by 3 (instead of factorising) would change the expression. However, in an equation, we can divide both sides by 3 without affecting the solutions.

Example

Solve the equation $3x^2 - 18x + 4 = 0$ by completing the square.

Exercise 2C/ 2D Page 23/ 24

Functions:



Transition Task. Chapter 1 - Algebraic Expressions.
Chapter 2 – Quadratics. Chapter 3 - Equations and Inequalities

Examples:

1. If $f(x) = x^2 - 3x$ and $g(x) = x + 5, \quad x \in \mathbb{R}$

   a) Find $f(-4)$

   b) Find the values of x for which $f(x) = g(x)$

   c) Find the roots of $f(x)$.

   d) Find the roots of $g(x)$.

2. Determine the minimum value of the function $f(x) = x^2 - 6x + 2$, and state the value of x for which this minimum occurs.

Test Your Understanding:

$f(x)$	**Completed square**	**Min/max value of $f(x)$**	**x for which this min/max occurs**
$x^2 + 4x + 9$			
$x^2 - 10x + 21$			
$10 - x^2$			
$8 - x^2 + 6x$			

1. Find the minimum value of $f(x) = 2x^2 + 12x - 5$ and state the value of x for which this occurs.

2. Find the roots of the function $f(x) = 2x^2 + 3x + 1$

3. Find the roots of the function $f(x) = x^4 - x^2 - 6$

Exercise 2E Page 27

Quadratic Graphs:

Example: Sketch the graph of $y = x^2 + 3x - 4$ and find the coordinates of the turning point.

Example

Sketch the graph of $y = 4x - 2x^2 - 3$ and find the coordinates of the turning point. Write down the equation of the line of symmetry.

Test Your Understanding

Sketch the following, indicating any intercepts with the axis, the turning point and the equation of the line of symmetry.

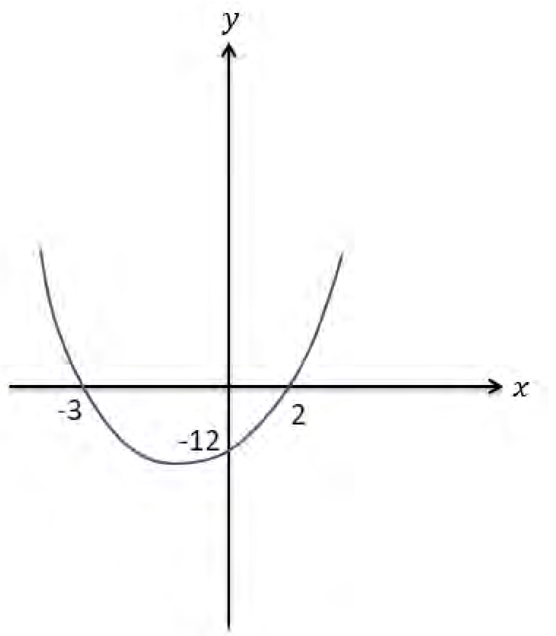
1. $y = x^2 + 4$

2. $y = x^2 - 7x + 10$

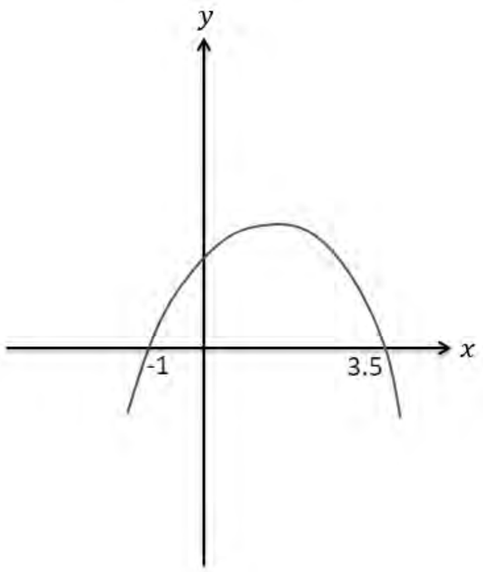
3. $y = 5x + 3 - 2x^2$

4. $y = x^2 + 4x + 11$

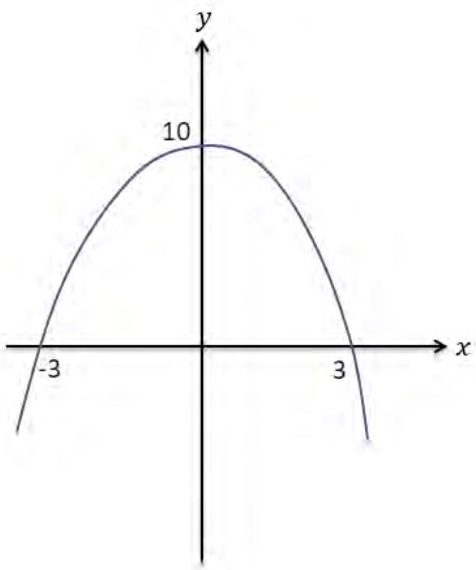
Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



Determine the equation of this quadratic graph in the form $y = ax^2 + bx + c$



Exercise 2F Page

The Discriminant

Quick fire questions:

Equation	Discriminant	No. of distinct real roots
$x^2 + 3x + 4 = 0$		
$x^2 - 4x + 1 = 0$		
$x^2 - 4x + 4 = 0$		
$2x^2 - 6x - 3 = 0$		
$x - 4 - 3x^2 = 0$		
$1 - x^2 = 0$		

Example:

8. The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(*a*) Find the value of p.

(4)

(*b*) For this value of p, solve the equation $x^2 + 2px + (3p + 4) = 0$.

(2)

Test Your Understanding:

1. $x^2 + 5kx + (10k + 5) = 0$ where k is a positive constant.

Given that this equation has equal roots, determine the value of k.

2. Find the range of values of k for which $x^2 + 6x + k = 0$ has two distinct real solutions.

Exercise 2G Page 32

Modelling

Example

A spear is thrown over level ground from the top of a tower.

The height, in metres, of the spear above the ground after t seconds is modelled by the function: $h(t) = 12.25 + 14.7t - 4.9t^2, \quad t \geq 0$

a) Interpret the meaning of the constant term 12.25 in the model.

b) After how many seconds does the spear hit the ground?

c) Write $h(t)$ in the form $A - B(t - C)^2$, where $A,\ B$ and C are constants to be found.

d) Using your answer to part c or otherwise, find the maximum height of the spear above the ground, and the time at which this maximum height is reached?

Exercise 2H Page 34

<u>Quadratics exam style question</u>

A ball is thrown upwards from a rooftop 80m above the ground. It will reach a maximum vertical height and then fall back to the ground.

The height of the ball from ground at time t is h, given by the formula:

$h = -16t^2 + 64t + 80$

a) Calculate the height reached by the ball after 1 second.

b) Calculate the maximum height reached by the ball and after how many seconds from when it is thrown this maximum height is reached.

c) Calculate how long will it take before the ball hits the ground.

Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.

p *x* *x* *p* *p*

p *p* *p*

p

$$x\sqrt{2} - \sqrt{18} = x$$

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

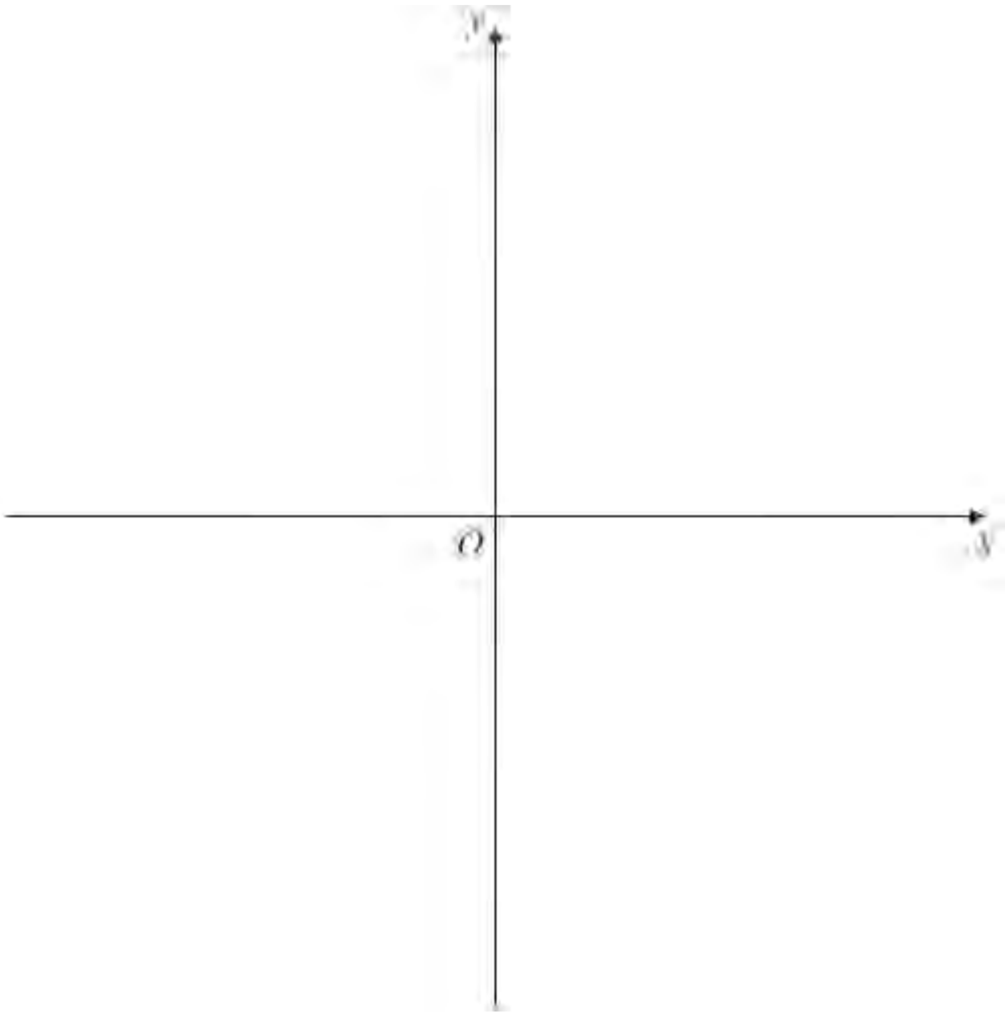
$$4x - 5 - x^2 = q - (x + p)^2$$

p q

p q

x x

y x x



$$f(x) = x^2 + (k+3)x + k$$

k

x *k*

x *k* *a* *b* *a*
b

k *x*

x *px* *p* *p*
p

Question Number	Scheme		Marks
(a)	$b^2-4ac<0 \Rightarrow$ e.g. $4^2-4(p-1)(p-5)<0$ or $0>4^2-4(p-1)(p-5)$ or $4^2<4(p-1)(p-5)$ or $4(p-1)(p-5)>4^2$	M1: Attempts to use b^2-4ac with at least two of *a*, *b* or *c* correct. May be in the quadratic formula. Could also be, for example, comparing or equating b^2 and $4ac$. Must be considering the given quadratic equation. Inequality sign not needed for this M1. There must be no *x* terms. A1: For a correct un-simplified **inequality** that is not the given answer	M1A1
	$4<p^2-6p+5$		
	$p^2-6p+1>0$	Correct solution with **no** errors that includes an expansion of $(p-1)(p-5)$	A1*
			(3)
(b)	$p^2-6p+1=0 \Rightarrow p=...$	For an attempt to solve $p^2-6p+1=0$ (**not <u>their</u> quadratic**) leading to 2 solutions for *p* (do not allow attempts to **factorise** – must be using the quadratic formula or completing the square)	M1
	$p=3\pm\sqrt{8}$	$p=3\pm2\sqrt{2}$ or any equivalent correct expressions e.g. $p=\frac{6\pm\sqrt{32}}{2}$ (May be implied by their inequalities) Discriminant must be a single number not e.g. 36 - 4	A1
	Allow the M1A1 to score anywhere for solving the given quadratic		
	$p<3-\sqrt{8}$ or $p>3+\sqrt{8}$	M1: Chooses outside region – **not dependent on the previous method mark** A1: $p<3-\sqrt{8}$, $p>3+\sqrt{8}$ or equivalent e.g. $p<\frac{6-\sqrt{32}}{2}$, $p>\frac{6+\sqrt{32}}{2}$ $(-\infty, 3-\sqrt{8})\cup(3+\sqrt{8}, \infty)$ Allow ",", "or" or a space between the answers but do **not** allow $p<3-\sqrt{8}$ **and** $p>3+\sqrt{8}$ (this scores M1A0) **Apply ISW if necessary.**	M1A1

A correct solution to the quadratic followed by $p > 3 \pm \sqrt{8}$ scores M1A1M0A0		
$3 + \sqrt{8} < p < 3 - \sqrt{8}$ scores M1A0		
Allow candidates to use x rather than p but must be in terms of p for the final A1		
		(4)
		(7 marks)

Question	Scheme	Marks	AOs
(i)	$x\sqrt{2} - \sqrt{18} = x \Rightarrow x\left(\sqrt{2} - 1\right) = \sqrt{18} \Rightarrow x = \dfrac{\sqrt{18}}{\sqrt{2} - 1}$	M1	1.1b
	$\Rightarrow x = \dfrac{\sqrt{18}}{\sqrt{2} - 1} \times \dfrac{\sqrt{2} + 1}{\sqrt{2} + 1}$	dM1	3.1a
	$x = \dfrac{\sqrt{18}\left(\sqrt{2} + 1\right)}{1} = 6 + 3\sqrt{2}$	A1	1.1b
		(3)	
(ii)	$4^{3x-2} = \dfrac{1}{2\sqrt{2}} \Rightarrow 2^{6x-4} = 2^{-\frac{3}{2}}$	M1	2.5
	$6x - 4 = -\dfrac{3}{2} \Rightarrow x = \ldots$	dM1	1.1b
	$x = \dfrac{5}{12}$	A1	1.1b
		(3)	
			(6 marks)

Notes

(i)

M1: Combines the terms in x, factorises and divides to find x. Condone sign slips and ignore any attempts to simplify $\sqrt{18}$
Alternatively squares both sides $x\sqrt{2}-\sqrt{18}=x \Rightarrow 2x^2-12x+18=x^2$

dM1: Scored for a complete method to find x. In the main scheme it is for making x the subject and then multiplying both numerator and denominator by $\sqrt{2}+1$
In the alternative it is for squaring both sides to produce a 3TQ and then factorising their quadratic equation to find x. (usual rules apply for solving quadratics)

A1: $x=6+3\sqrt{2}$ only following a correct intermediate line. Allow $\frac{6+3\sqrt{2}}{1}$ as an intermediate line.
In the alternative method the $6-3\sqrt{2}$ must be discarded.

(ii)

M1: Uses correct mathematical notation and attempts to set both sides as powers of 2 or 4.
Eg $2^{ax+b}=2^c$ or $4^{dx+e}=4^f$ is sufficient for this mark.
Alternatively uses logs (base 2 or 4) to get a linear equation in x.
$4^{3x-2}=\frac{1}{2\sqrt{2}} \Rightarrow \log_2 4^{3x-2}=\log_2\frac{1}{2\sqrt{2}} \Rightarrow 2(3x-2)=\log_2\frac{1}{2\sqrt{2}}$.
Or $4^{3x-2}=\frac{1}{2\sqrt{2}} \Rightarrow 3x-2=\log_4\frac{1}{2\sqrt{2}}$

Or $4^{3x-2}=\frac{1}{2\sqrt{2}} \Rightarrow 4^{3x}=4\sqrt{2} \Rightarrow 3x=\log_4 4\sqrt{2}$

dM1: Scored for a complete method to find x.
Scored for setting the indices of 2 or 4 equal to each other and then solving to find x.
There must be an attempt on both sides.

You can condone slips for this mark Eg bracketing errors $4^{3x-2} = 2^{2\times 3x-2}$ or $\frac{1}{2\sqrt{2}} = 2^{-1+\frac{1}{2}}$

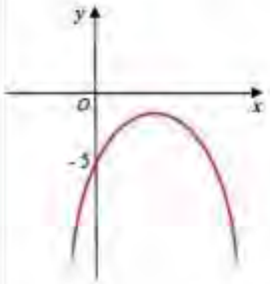
In the alternative method candidates cannot just write down the answer to the rhs.

So expect some justification. E.g. $\log_2 \frac{1}{2\sqrt{2}} = \log_2 2^{-\frac{3}{2}} = -\frac{3}{2}$

or $\log_4 \frac{1}{2\sqrt{2}} = \log_4 2^{-\frac{3}{2}} = -\frac{3}{2} \times \frac{1}{2}$ condoning slips as per main scheme

or $3x = \log_4 4\sqrt{2} \Rightarrow 3x = 1 + \frac{1}{4}$

A1: $x = \frac{5}{12}$ with correct intermediate work

Question Number	Scheme	Marks
	$4x-5-x^2 = q-(x-p)^2$, p, q are integers.	
(a)	$\{4x-5-x^2 =\}\ -[x^2-4x+5] = -[(x-2)^2-4+5] = -[(x-2)^2+1]$	M1
	$= -1-(x-2)^2$	A1 A1
		[3]
(b)	$\{"b^2-4ac" =\}\ 4^2-4(-1)(-5)\quad \{=16-20\}$	M1
	$= -4$	A1
		[2]
(c)		
	Correct ∩ shape	M1
	Maximum within the 4th quadrant	A1
	Curve cuts through -5 or $(0,-5)$ marked on the y-axis	B1
		[3]
		8

Notes

(a)

M1: for an attempt to complete the square eg: $\pm(\pm x \pm 2)^2 \pm k - 5,\ k \neq 0$ or $\pm(\pm x \pm 2)^2 \pm \lambda,\ \lambda \neq -5$ seen or implied in working

1st A1: for $p = -2$ or for $\pm\alpha-(x-2)^2$, α can be 0.

2nd A1: for $q = -1$

Note: Allow M1A1A1 for a correct written down expression of $-1-(x-2)^2$ Ignore $-1-(x-2)^2 = 0$.

Note: If a candidate states either $p=-2$ or $q=-1$ or writes $\pm k-(x-2)^2$ then imply the M1 mark.

Note: A candidate who writes down with no working $p=2$, $q=$ (a value which is not -1) gets M0A0A0.

Note: Writing $(x-2)^2-1$, followed by $p=-2$, $q=-1$ is M1A1A0.

Alternative 1 to (a)

$\{4x-5-x^2 =\}\ -[x^2-4x]-5 = -[(x-2)^2-4]-5 = -(x-2)^2+4-5 = -1-(x-2)^2$

Alternative2 to (a)

$q-(x+p)^2 = q-(x^2+2px+p^2) = -x^2-2px+q-p^2$

Compare x terms: $-2p = 4 \Rightarrow \underline{p=-2}$

Compare constant terms: $q-p^2=-5 \Rightarrow q-4=-5 \Rightarrow \underline{q=-1}$

M1: Either $\pm 2p = 4$ or $q \pm p^2 = -5$; **1st A1:** for $p=-2$; **2nd A1:** for $q=-1$

Alternative 3 to (a)

Negating $4x-5-x^2$ gives x^2-4x+5

So, $x^2-4x+5 = (x-2)^2-4+5\ \{=(x-2)^2+1\}$ **M1** for $\pm(\pm x \pm 2)^2 \pm k + 5$

then stating $p=-2$ is **1st A1** and/or $q=-1$ is **2nd A1**.

or writing $-1-(x-2)^2$ is A1A1.

Special Case for part (a):

$q-(x+p)^2 = q-(x^2+2px+p^2) = -x^2-2px+q-p^2 = 4x-5-x^2$

$\Rightarrow -2px+q-p^2 = 4x-5 \Rightarrow q-p^2+5 = 4x+2px \Rightarrow q-p^2+5 = x(4+2p)$

$\Rightarrow x = \dfrac{q-p^2+5}{4+2p} \Rightarrow p \neq -2$ scores Special Case M1A1A1 **only once** $p \neq -2$ achieved

(b)

M1: for correctly substituting any two of $a=-1$, $b=4$, $c=-5$ into b^2-4ac if this is quoted.

If b^2-4ac is not quoted then the substitution must be correct.

Substitution into $b^2 < 4ac$ or $b^2 = 4ac$ or $b^2 > 4ac$ is M0.

A1: for -4 only

If they write $-4 < 0$ treat the < 0 as ISW and award A1. If they write $-4 \geq 0$ then score A0.

So substituting into $b^2-4ac<0$ leading to $-4<0$ can score M1A1

Note: Only award marks for use of the discriminant in part (b).

Note: Award M0 if the candidate uses the quadratic formula UNLESS they later go on to identify that the discriminant is the result of b^2-4ac.

Beware: A number of candidates are writing up their solution to part (b) at the bottom of the second page. So please look!

(c)

M1: Correct ∩ shape in any quadrant.

A1: The maximum must be *within* the fourth quadrant to award this mark.

B1: Curve *(and not line!)* cuts through -5 or $(0,-5)$ marked on the y-axis

Allow $(-5, 0)$ rather than $(0,-5)$ if marked in the "correct" place on the y-axis.

If the curve cuts through the negative y-axis and this is not marked, then you can recover $(0,-5)$ from the candidate's working in part (c). You are not allowed to recover this point, though, from a table of values.

Note: Do not recover work for part (a) in part (c).

Question Number	Scheme	Marks
(a)	Discriminant: $b^2-4ac=(k+3)^2-4k$ or equivalent	M1 A1 (2)
(b)	$(k+3)^2-4k=k^2+2k+9=(k+1)^2+8$	M1 A1 (2)
(c)	For real roots, $b^2-4ac\geq 0$ or $b^2-4ac>0$ or $(k+1)^2+8>0$ $(k+1)^2\geq 0$ for all k, so $b^2-4ac>0$, so roots are real for all k (or equiv.)	M1 A1 cso (2) **6**
	Notes (a) M1: attempt to find discriminant – substitution is required If formula b^2-4ac is seen at least 2 of a, b and c must be correct If formula b^2-4ac is **not** seen all 3 of a, b and c must be correct Use of b^2+4ac is M0 A1: correct unsimplified (b) M1: Attempt at completion of square (see earlier notes) A1: both correct (no ft for this mark) (c) M1: States condition as on scheme **or** attempts to explain that their $(k+1)^2+8$ is greater than 0 A1: The final mark (A1cso) requires $(k+1)^2\geq 0$ and conclusion. We will allow $(k+1)^2>0$ (or word positive) also allow $b^2-4ac\geq 0$ and conclusion.	

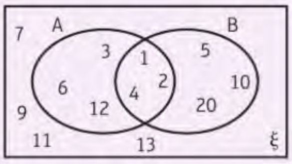
Question Number	Scheme	Marks
Q	b^2-4ac attempted, in terms of p. $(3p)^2-4p=0$ o.e. Attempt to solve for p e.g. $p(9p-4)=0$ Must potentially lead to $p=k,\ k\neq 0$ $p=\frac{4}{9}$ (Ignore $p=0$, if seen)	M1 A1 M1 A1cso **[4]**
	1st M1 for an attempt to substitute into b^2-4ac or $b^2=4ac$ with b or c correct Condone x's in one term only. This can be inside a square root as part of the quadratic formula for example. **Use of inequalities can score the M marks only** 1st A1 for any correct equation: $(3p)^2-4\times1\times p=0$ or better 2nd M1 for an attempt to factorize or solve their quadratic expression in p. Method must be sufficient to lead to their $p=\frac{4}{9}$. Accept factors or use of quadratic formula or $(p\pm\frac{2}{9})^2=k^2$ (o.e. eg) $(3p\pm\frac{2}{3})^2=k^2$ or equivalent work on <u>their</u> eqn. $9p^2=4p\Rightarrow\frac{9p^2}{p}=4$ which would lead to $9p=4$ is OK for this 2nd M1 ALT <u>Comparing coefficients</u> M1 for $(x+\alpha)^2=x^2+\alpha^2+2\alpha x$ and A1 for a correct equation eg $3p=2\sqrt{p}$ M1 for forming solving leading to $\sqrt{p}=\frac{2}{3}$ or better <u>Use of quadratic discriminant formula (or any formula) Rule for awarding M mark</u> If the formula is quoted accept some correct substitution leading to a partially correct expression. If the formula is not quoted only award for a fully correct expression using their values.	

Diagnostic for Chapter 3 Equations and Inequalities

1 A = {factors of 12}

B = {factors of 20}

Write down the numbers in each of these sets:



a $A \cap B$ **b** $(A \cup B)'$

2 Simplify these expressions.

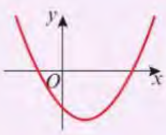
a $\sqrt{75}$ **b** $\dfrac{2\sqrt{45} + 3\sqrt{32}}{6}$

3 Match the equations to the correct graph. Label the points of intersection with the axes and the coordinates of the turning point.

a $y = 9 - x^2$ **b** $y = (x - 2)^2 + 4$

c $y = (x - 7)(2x + 5)$

i



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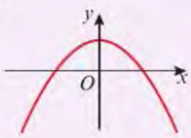

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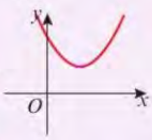


Simultaneous Equations

Simultaneous Equations Solution Sets

Scenario	Example	Solution Set
A single solution:	$x + y = 9$ $x - y = 1$	
Two solutions:	$x^2 + y^2 = 10$ $x + y = 4$	
No solutions:	$x + y = 1$ $x + y = 3$	
Infinitely large set of solutions:	$x + y = 1$ $2x + 2y = 2$	

Example (You can do this on your calculator!)

Solve the simultaneous equations

$$3x + y = 8$$

$$2x - 3y = 9$$

Method 1 : Elimination

Method 2: Substitution

Exercise 3A Page 40

Linear and Quadratic

Example:

Solve the simultaneous equations:

$$x + 2y = 3$$

$$x^2 + 3xy = 10$$

Test Your Understanding:

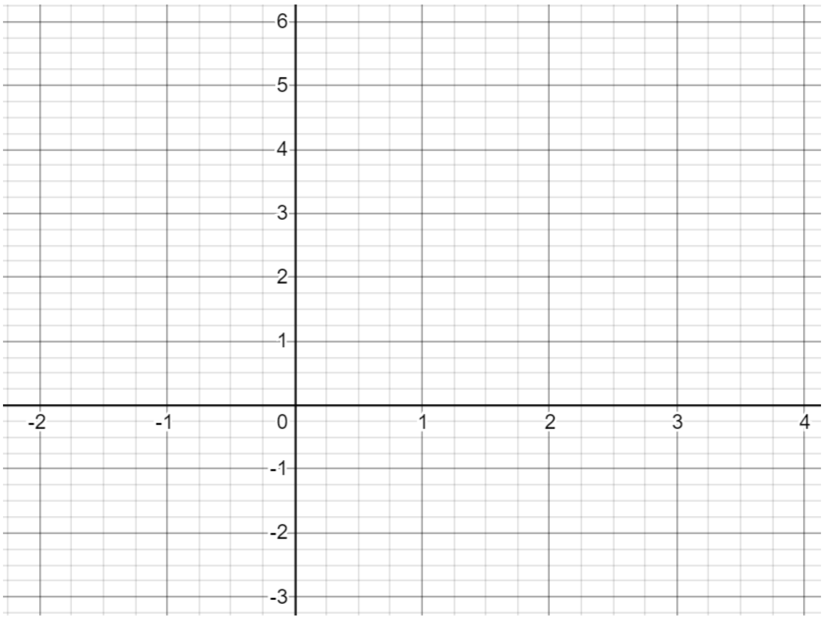
1. Solve the simultaneous equations: $3x^2 + y^2 = 21$ and $y = x + 1$

Exercise 3B Page 41

Simultaneous Equations and Graphs

Examples:

1a. On the same axes, draw the graphs of $2x + y = 3$ and $y = x^2 - 3x + 1$



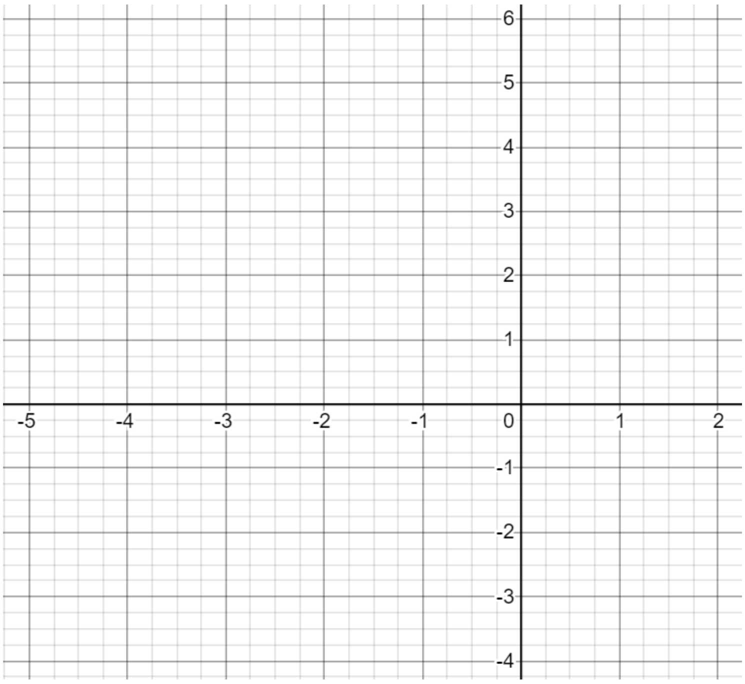
1b. Use your graph to write down the solutions to the simultaneous equations

1c. What algebraic method could we have used to show the graphs would have intersected twice?

Example 2

a) On the same axes, draw the graphs of:

$$y = 2x - 2 \qquad y = x^2 + 4x + 1$$



b) Prove algebraically that the lines never meet

Test Your Understanding

The line with equation $y = 2x + 1$ meets the curve with equation $kx^2 + 2y + (k - 2) = 0$ at exactly one point. Given that k is a positive constant:

a) Find the value of k.

b) For this value of k, find the coordinates of this point of intersection

Exercise 3C Page 45

Set Builder Notation

Recap from GCSE:

We use curly braces to list the values in a set, e.g. $A = \{1,4,6,7\}$
If A and B are sets then $A \cap B$ is the **intersection** of A and B, giving a set which has the elements in A **and** B.
$A \cup B$ is the **union** of A and B, giving a set which has the elements in A **or** in B.
∅ is the empty set, i.e. the set with nothing in it.
Sets can also be infinitely large. $\mathbb{N}$ is the set of natural numbers (all positive integers), $\mathbb{Z}$ is the set of all integers (including negative numbers and 0) and $\mathbb{R}$ is the set of all real numbers (including all possible decimals).
We write $x \in A$ to mean "x is a member of the set A". So $x \in \mathbb{R}$

Quick Fire Examples

$\{1,2,3\} \cap \{3,4,5\} =$

$\{1,2,3\} \cup \{3,4,5\} =$

$\{1,2\} \cap \{3,4\} \quad =$

Examples:

1. $\{2x : x \in \mathbb{Z}\}$

2. $\{2^x : x \in \mathbb{N}\}$

3. $\{xy:\ x, y\ are\ prime\}$

Solving Inequalities

Linear inequalities Examples

1. $2x + 1 > 5$

2. $3(x - 5) \geq 5 - 2(x - 8)$

3. $-x \geq 2$

Combining Inequalities

When combining inequalities always draw a number line to help!

Example:

If $x < 3$ and $2 \leq x < 4$, what is the combined solution set?

Quadratic Inequalities:

Examples

1. Solve $x^2 + 2x - 15 > 0$

2. Solve $x^2 + 2x - 15 \leq 0$

3. Solve $x^2 + 5x \geq -4$

4. Solve $x^2 < 9$

Test Your Understanding

Find the set of values of x for which

(*a*) $3(x-2) < 8-2x$, **(2)**

(*b*) $(2x-7)(1+x) < 0$, **(3)**

(*c*) both $3(x-2) < 8-2x$ **and** $(2x-7)(1+x) < 0$. **(1)**

Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(*a*) show that $q^2 + 8q < 0$. **(2)**

(*b*) Hence find the set of possible values of q. **(3)**

Division by x

Find the set of values for which $\frac{6}{x} > 2, \quad x \neq 0$

Exercise 3D/ 3E Page 47 – 51

<u>Sketching Inequalities:</u>

Examples

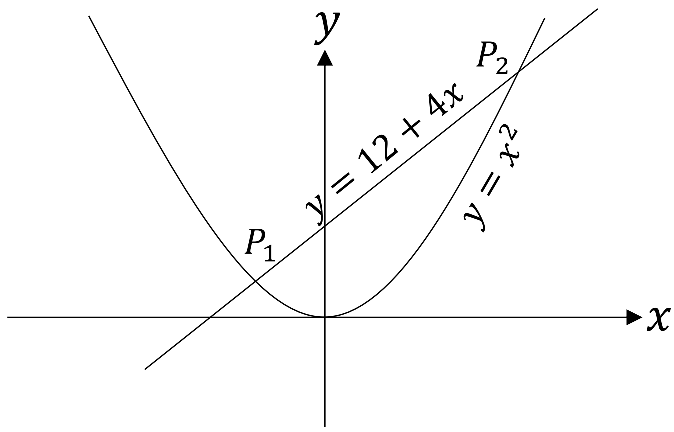
1. L_1 has equation $y = 12 + 4x$. L_2 has equation $y = x^2$.

The diagram shows a sketch of L_1 and L_2 on the same axes.

a) Find the coordinates of P_1 and P_2, the points of intersection.

b) Hence write down the solution to the inequality

$12 + 4x > x^2$.



2. Shade the region that satisfies the inequalities:

$$2y + x < 14$$

$$y \geq x^2 - 3x - 4$$

Need a recap of the content in this chapter? Use this QR code to watch a Bicen maths YouTube video.

Exercise 3F/ 3G Page 53 – 55

x

x *x*

x *x*

x

x *x*

x *x*

x *x* *x* *x*

k *x* *x* *k* *k*

x

k

k *k*

k

x

x *x*

x *x*

x *x* *x* *x*

Question Number	Scheme		Marks
(a)	$6x + x > 1 - 8$	Attempts to expand the bracket and collect x terms on one side and constant terms on the other. Condone sign errors and allow one error in expanding the bracket. Allow $<, \leq, \geq, =$ instead of $>$.	M1
	$x > -1$	Cao	A1
	Do not isw here, mark their final answer.		
			(2)
(b)	$(x+3)(3x-1)[=0]$ $\Rightarrow x = -3$ and $\frac{1}{3}$	M1: Attempt to solve the quadratic to obtain two critical values A1: $x = -3$ **and** $\frac{1}{3}$ (may be implied by their inequality). Allow all equivalent fractions for -3 and 1/3. (Allow 0.333 for 1/3)	M1A1
	$-3 < x < \frac{1}{3}$	M1: Chooses "inside" region (The letter x does not need to be used here) A1ft: Allow $x < \frac{1}{3}$ **and** $x > -3$ or $\left(-3, \frac{1}{3}\right)$ or $x < \frac{1}{3} \cap x > -3$. Follow through their critical values. (must be in terms of x here) Allow all equivalent fractions for -3 and 1/3. Both ($x < \frac{1}{3}$ **or** $x > -3$) and ($x < \frac{1}{3}$, $x > -3$) as a final answer score A0.	M1A1ft
			(4)
			[6]
	Note that use of $\leq$ or $\geq$ appearing in an otherwise correct answer in (a) or (b) should only be penalised once, the first time it occurs.		

Question Number	Scheme	Marks
Q (a)	$5x > 10$, $x > 2$ [Condone $x > \frac{10}{5} = 2$ for M1A1]	M1, A1 (2)
(b)	$(2x+3)(x-4) = 0$, 'Critical values' are $-\dfrac{3}{2}$ and 4	M1, A1
	$-\dfrac{3}{2} < x < 4$	M1 A1ft (4)
(c)	$2 < x < 4$	B1ft (1) [7]
(a)	M1 for attempt to collect like terms on each side leading to $ax > b$, or $ax < b$, or $ax = b$ Must have a or b correct so eg $3x > 4$ scores M0	
(b)	1st M1 for an attempt to factorize or solve to find critical values. Method must potentially give 2 critical values 1st A1 for $-\dfrac{3}{2}$ and 4 seen. They may write $x < -\dfrac{3}{2}$, $x < 4$ and still get this A1 2nd M1 for choosing the "inside region" for their critical values 2nd A1ft follow through their 2 distinct critical values Allow $x > -\dfrac{3}{2}$ with "or" "," "$\cup$" " " $x < 4$ to score M1A0 but "and" or "$\cap$" score M1A1 $x \in (-\frac{3}{2}, 4)$ is M1A1 but $x \in [-\frac{3}{2}, 4]$ is M1A0. Score M0A0 for a number line or graph only	
(c)	B1ft Allow if **a correct answer is seen** or follow through their answer to (a) and their answer to (b) but their answers to (a) and (b) <u>must be regions</u>. Do not follow through single values. If their follow through answer is the empty set accept $\varnothing$ or {} or equivalent in words If (a) or (b) are not given then score this mark for cao NB You may see $x<4$ (with anything or nothing in-between) $x < -1.5$ in (b) and empty set in (c) for B1ft **Do not award marks for part (b) if only seen in part (c)** Use of $\leq$ instead of $<$ (or $\geq$ instead of $>$) loses one accuracy mark only, at first occurrence.	

Question Number	Scheme	Marks
(a)	**Method 1:** Attempts $b^2 - 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ $b^2 - 4ac = 6^2 - 4(k+3)(k-5)$ $(b^2 - 4ac =)$ $-4k^2 + 8k + 96$ or $-(b^2 - 4ac =)$ $4k^2 - 8k - 96$ (with no prior algebraic errors) As $b^2 - 4ac > 0$, then $-4k^2 + 8k + 96 > 0$ and so, $k^2 - 2k - 24 < 0$	M1 A1 B1 A1 *
	Method 2: Considers $b^2 > 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ $6^2 > 4(k+3)(k-5)$ $4k^2 - 8k - 96 < 0$ or $-4k^2 + 8k + 96 > 0$ or $9 > (k+3)(k-5)$ (with no prior algebraic errors) and so, $k^2 - 2k - 24 < 0$ following correct work	M1 A1 B1 A1 * **[4]**
(b)	Attempts to solve $k^2 - 2k - 24 = 0$ to give $k =$ ($\Rightarrow$ Critical values, $k = 6, -4$.) $k^2 - 2k - 24 < 0$ gives $-4 < k < 6$	M1 M1 A1 **[3]** **7 marks**
	Notes	
(a)	**Method 1: M1:** Attempts $b^2 - 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ or uses quadratic formula and has this expression under square root. (ignore > 0, < 0 or $= 0$ for first 3 marks) **A1:** Correct expression for $b^2 - 4ac$ – need not be simplified (may be under root sign) **B1:** Uses algebra to manipulate result **without error** into **one of these** three term quadratics. Again may be under root sign in quadratic formula. If inequality is used early in "proof" may see $4k^2 - 8k - 96 < 0$ and B1 would be given for $4k^2 - 8k - 96$ correctly stated. **A1:** Applies $b^2 - 4ac > 0$ correctly (**or writes** $b^2 - 4ac > 0$) to achieve the **result given** in the question. No errors should be seen. Any incorrect line of argument should be penalised here. There are several ways of reaching the answer; either multiplication of both sides of inequality by -1, or taking every term to other side of inequality. **Need conclusion i.e. printed answer.** **Method 2: M1:** Allow $b^2 > 4ac$ $b^2 < 4ac$ or $b^2 = 4ac$ for $a = (k+3)$, $b = 6$ and their c. $c \neq k$ **A1:** Correct expressions on either side (ignore $>$, $<$ or $=$) **B1:** Uses algebra to manipulate result into one of the two three term quadratics or divides both sides by 4 **again without error** **A1:** Produces result with no errors seen from initial consideration of $b^2 > 4ac$.	
(b)	**M1:** Uses factorisation, formula, completion of square method to find two values for k, or finds two **correct** answers with no obvious method **M1:** Their Lower Limit $< k <$ Their Upper Limit Allow the M mark mark for $\leq$ (Allow $k <$ upper and $k >$ lower) **A1:** $-4 < k < 6$ Lose this mark for $\leq$ Allow $(-4, 6)$ [not square brackets] or $k > -4$ **and** $k < 6$ (must be **and** not or) Can also use intersection symbol $\cap$ **NOT** $k > -4$, $k < 6$ (M1A0)	
	Special case : **In part (a)** uses $c = k$ instead of $k - 5$ - scores 0 . Allow $k + 5$ for method marks	
	Special Case: In part (b) Obtaining $-6 < k < 4$ This is a common wrong answer. Give M1 M1 A0 special case	
	Special Case: In part (b) Use of x instead of k – M1M1A0	
	Special Case: $-4 < k < 6$ and $k < -4$, $k > 6$ both given is M0A0 for last two marks. Do not treat as isw.	

Question Number	Scheme	Marks
	(a) $3x-7>3-x$	
	$4x>10$	M1
	$x>2.5,\quad x>\frac{5}{2},\quad \frac{5}{2}<x$ o.e.	A1
		(2)
	(b) Obtain $x^2-9x-36$ and attempt to solve $x^2-9x-36=0$	
	e.g. $(x-12)(x+3)=0$ so $x=$, or $x=\frac{9\pm\sqrt{81+144}}{2}$	M1
	12, –3	A1
	$-3\le x\le 12$	M1A1
		(4)
	(c) $2.5<x\le 12$	A1cso
		(1)
		(7 marks)

px *q* *p* *q* *x*
x
x

x *x* *x*
x *x* *x* *x*
x

x *x*
x *x* *x* *x* *x*

x *x*